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CASCADE CHARACTERISTIC NUMBERS FOR TWO-DIMENSIONAL COMPRESSIBLE FLOW

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CASCADE CHARACTERISTIC NUMBERS FOR TWO-DIMENSIONAL COMPRESSIBLE FLOW

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FOREWORD

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ABSTRACT

Several basic methods for evaluating cascade characteristics are investigated from the standpoint of their completeness in describing the cascade flow. Characteristic numbers are defined that directly relate the properties of the cascade with the properties of a corresponding compressor. Compressibility is taken into account, which enables the application of the characteristic numbers for transonic cascade flow. The relationships between the characteristic numbers are determined.

This report does not treat the problems involved with establishing two-dimensional flow in the cascade, instrumentation and measuring techiques; rather, basically it is assumed that two-dimensional cascade flow is realized.

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Introduction:

The purpose of two-dimensional cascade research work as considered in this paper is to obtain basic information for the design of turbomachinery. Specifically, the object in two-dimensional cascade research work is to determine relationships between the cascade variables and parameters, and the pressure rise, the flow deflection and the losses through the cascade. The most significant parameters of two-dimensional cascades are the following:

A. Blade Parameters and Variables

Thickness distribution
Camber
Nose radius
Surface roughness
Boundary layer removal
Slots
Porosity (suction)

B. Channel Parameters and Variables

Inflow-outflow angle
Stagger
Spacing
End Effect (boundary layer removal)
Velocity (pressure) gradients

C. Fluid Parameters and Variables

Mach number
Reynolds number
Turbulence

In a given compressor stage (as a whole) the blade elements at different radii cannot be considered as isolated from each other because of the three-dimensional flow effects. Thus, in general, the two-dimensional cascade tests results will not provide a sufficient basis for the design of turbomachines. However information about many basic problems which are important for the design of axial flow compressors can be gained from such tests.

Some of the typical problems are:

1) Limitations of the intake Mach number and blade load for

various cascade configurations.

2) Deviation of the flow angle from the tangent of the mean camber line at the trailing edge.

3) Influence of the Reynolds number.

. 4) Influence of the blade shapes and cascade geometry upon the limiting intake Mach number.

In order to determine the cascade flow, usually some of the following measurements are taken:

1) Intake conditions into the cascade; that is, intake speed, intake angle, intake stagnation conditions of the fluid medium.

2) Exit conditions; that is, distribution of static and stagnation pressure and flow direction.

3) In some cases, pressure distribution around a profile, and blade force measurements.

Generally the intake conditions are varied and the conditions under 2) or 3) are determined as functions of the intake conditions. The parameters determining the cascade are profile form, stagger angle, spacing ratio and aspect ratio.

In order to present the cascade test results in a general form, cascade characteristic numbers commonly are used. There are various systems of cascade characteristic numbers in use. Some refer to the drag and lift forces of the blades, corresponding to the characteristic numbers of single airfoils. The cascade drag and lift coefficients are practical if the aim of the cascade investigation is to make a comparison between cascade theory and an actual cascade flow. Furthermore they are useful for the design of turbomachines with relatively large blade spacing ratios and neglegible small changes of density through the cascade.

Other definitions of cascade characteristic numbers relate directly the pressure increase or the losses through the cascade with the velocity pressure of the intake velocity, W_1 or the mean velocity, W_m . However these definitions are not practical if the density change through the cascade is not neglegible.

It is the purpose of this paper to investigate cascade characteristic numbers which are applicable for incompressible and compressible flow, and to determine their mutual relationships, and their relationships to the commonly accepted characteristic numbers for axial flow turbomachines.

I.   Definitions of Cascade Characteristic Numbers

Two basic difficulties are encountered with two-dimensional cascade investigations. The one results from the fact that wakes behind the cascade blades cause a non-uniform distribution of total pressure and entropy of the flow field. The other is due to tunnel wall interference which causes a deviation from two-dimensional flow and a superimposed drop of static pressure in the direction of the flow due to the skin friction of the tunnel walls.

The influence of the tunnel walls on the cascade flow can be diminished by the following means:

a)   Application of a large blade aspect ratio or of wall boundary layer suction; thereby the deviations from two-dimensional flow will be decreased.

b)   Application of a large number of blades. Both increasing the blade aspect ratio and increasing the number of blades result in an increase of the tunnel dimensions in comparison to the blade chord length. Thereby the superimposed drop in static pressure due to the skin friction at the tunnel walls behind the cascade decreases, since the furthest distance between the measuring stations and cascade is a certain multiple of the blade chord length.

c)   If straight tunnel side walls are applied, then the pressure integral over the surface of the side walls has to be the same for both side walls in order to insure that the side walls do not contribute to the flow deflection.

The means mentioned under point a) and b) increase considerably the costs of the tunnel and of the cascade tests installations. Hence, some types of cascade investigations using smaller tunnels, where interference effects between the tunnel walls and the cascade flow are unavoidable, may be justified for the determination of relative values and trends. However for the following derivation of the inter-relationships between the cascade characteristic numbers and the relationships between the compressor characteristic numbers, tunnel wall interference effects will be neglected.

The flow behind the cascade can be described as follows: Immediately behind the cascade the total pressure and entropy are nearly uniform in the main stream between the blades, and drop very suddenly in the region of the wakes behind the profiles. Further downstream the wakes disperse and the flow becomes uniform whereby the static pressure increases. The momentum transport of the flow downstream the cascade remains constant. By the effect of the dissipation of the wakes the direction of the mean

streamline changes. Further, the integral of the total pressure over the
mass flow decreases, while the integral of the entropy over the mass flow
increases. This is because the entropy of the flow with constant momentum
transport becomes a maximum when the flow is uniform.

This consideration shows that all flow parameters down stream of the
cascade, except the momentum transport of the flow, vary between two limit-
ing conditions, namely the flow conditions immediately behind the cascade
and the flow conditions in such a distance where the wakes are dispersed
and uniform flow is established. Hence in order to evaluate the tests it
is important to know the distance between the measuring stations and the
cascade. Theoretical and practical investigations to determine the differ-
ence of the flow parameters for these two limiting conditions are recommended.

In a compressor composed of alternating rotating and stationary blade
rows the distance between the rows is usually so small that uniform flow
will not develop between the rows. Non-uniform flow distribution behind a
rotating blade row results in an unsteady flow for the following stationary
blade row and vice versa.

Thus in order to obtain flow conditions for a cascade similar to the
flow conditions in a compressor, unsteady flow upstream of the cascade
would be required. Although studying these effects appears to be very
important, in this paper only steady cascade flow will be treated. This
corresponds to a compressor with such a distance between the blade rows
that nearly uniform flow results between the blade rows. Hence the analysis
of the relationships between the characteristic numbers of cascade and com-
pressor and further of the inter-relationships between the characteristic
cascade numbers will be confined to the previously discussed limiting case
of the uniform flow behind the cascade without interference between cascade
and tunnel walls.

The principle requirements for the definitions of the characteristic
cascade numbers is that the velocity triangle of the cascade flow is com-
pletely determined. Thereby it will be possible to reconstruct the velocity
triangle of the cascade flow when the intake conditions and the values of
the characteristic cascade numbers are given. Further it is possible to
express by these characteristic cascade numbers any otherwise defined
cascade number.

For representing the pressure increase through the cascade, (see
figure (1), a pressure coefficient, K_p is defined. One definition fre-
quently used, is:

$$K_p = \frac{\Delta p_{1-2}}{\rho \frac{w_1^2}{2}} \quad \text{----------------} \quad (1)$$

This expression represents for the limiting case that the Mach number approaches zero the quotient of two energy terms, namely the isentropic change in enthalpy through the cascade, see figure (2), divided by the kinetic energy of the flow upstream the cascade.

In order that simple relationships will result between loss and pressure coefficients, and further between these coefficients and the compressor efficiency, this quotient of the two energy terms should be defined as the pressure coefficient without the restriction to small Mach numbers. This results in the following definition for K_ρ: compare figure (2).

$$K_\rho = \frac{\left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} - 1}{\left(\frac{p_0}{p_1}\right)^{\frac{\gamma-1}{\gamma}} - 1} = \frac{\left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} - 1}{\frac{\gamma-1}{2} M_1^2} \quad - - - - - - - - - \quad (2)$$

This expression for K_ρ goes over into that of equation (1) when the Mach number goes to zero which means that the flow can be considered as incompressible.

The loss coefficient, K_L, of the cascade is defined as the actual enthalpy change through the cascade minus the corresponding isentropic enthalpy change through the cascade, divided by the enthalpy change within the inlet bell*, see figure (2), thus K_L can be expressed as follows, also see figure (2).

$$K_L = \frac{\Delta T_{1-2} - \Delta T_{i,1-2}}{T_{t,1} - T_1} = \frac{\left(\frac{p_2}{p_0}\right)^{\frac{\gamma-1}{\gamma}}\left[\left(\frac{p_0}{p_{t,2}}\right)^{\frac{\gamma-1}{\gamma}} - 1\right]}{1 - \left(\frac{p_1}{p_0}\right)^{\frac{\gamma-1}{\gamma}}} \sim \frac{\Delta S_{1-2}}{c_p} \frac{T_2}{T_{t,1} - T_1} \quad - - - - \quad (3)$$

The entropy expression in equation (3) represents a good approximation as long as $\Delta T_{1-2} - \Delta T_{i,1-2}$ is very small in comparison to T_2. If this condition is not fulfilled, then $\frac{1}{2}(\Delta T_{1-2} - \Delta T_{i,1-2})$ should be subtracted from T_2 in equation (3), resulting in the following equation:

$$K_L = \frac{2 \Delta S_{1-2}}{2c_p + \Delta S_{1-2}} \cdot \frac{T_2}{T_{t,1} - T_1}$$

For small Mach numbers (incompressible flow the expression for K_L goes over into the ratio of total pressure drop through the cascade divided by the

*) Isentropic uniform flow upstream the cansade will be assumed throughout this report.

velocity pressure ahead the cascade.

$$\text{that is} \qquad K_l = \frac{2\Delta P_{t,l-2}}{\rho W_l^2} \qquad - - - - - - - - - - - - - \text{(3a)}$$

For representing the flow deflection, the turning angle, $\Delta\beta$ can be used, see figure 1.

$$\Delta\beta = \beta_2 - \beta_l \qquad - - - - - - - - - - - - - - \text{(4)}$$

The values of these coefficients derived from the cascade test may be presented diagrammatically as functions of the intake angle, the flow parameters, and the cascade geometry. As an example, a form for such a diagram is given in figure (3), wherein the intake angle, β, is taken as the variable. In general, the choice between variable and curve parameter is dependent upon the primary purpose of the investigation. For instance; if for a particular cascade configuration the influence of the flow parameters is to be determined, then it would be practical to use the Mach or Reynolds number as a variable.

Such a presentation completely describes the conditions of the flow upstream and downstream of the cascade. Thus it is possible to reconstruct the velocity triangle of the cascade flow when the pressure coefficient, K_p, the loss coefficient, K_l, the turning angle, $\Delta\beta$, and the intake conditions β_l; M_l are given.

Between the intake flow conditions, the turning angle, the loss, and the pressure coefficient, the following relationship exists:

$$I = \frac{\sin(\beta_l + \Delta\beta)(1 + K_p \frac{\gamma-1}{2} M_l^2)^{\frac{\gamma-1}{\gamma}} \sqrt{1 - K_l - K_p}}{\sin\beta_l \left(1 + (K_p + K_l) \frac{\gamma-1}{2} M_l^2\right)} \qquad - - - - - \text{(5)}$$

or for the limiting case of incompressible flow

$$I = \frac{\sin(\beta_l + \Delta\beta)}{\sin\beta_l} \sqrt{1 - K_l - K_p} \qquad - - - - - - - - \text{(5a)}$$

This equation can be derived as follows:

From the continuity condition it follows that

$$A_1 \, \rho_1 \, W_1 = A_2 \, \rho_2 \, W_2$$

or
$$1 = \frac{A_2}{A_1} \cdot \frac{W_2}{W_1} \cdot \frac{\rho_2}{\rho_1} \quad \text{------------------ (6)}$$

In this equation, $\frac{A_2}{A_1}$ can be replaced by $\frac{\sin(\beta_1 + \Delta\beta)}{\sin\beta_1}$, see figure (1).
Furthermore, $\frac{W_2}{W_1}$ can be replaced by $\sqrt{1 - K_L - K_\rho}$ which can be seen
as follows:

$$\Delta T_{i, 1-2} = T_1 \left[\left(\frac{P_2}{P_1} \right)^{\frac{\gamma-1}{\gamma}} - 1 \right] \\[2mm]
T_{t,1} - T_1 = T_1 \left[\left(\frac{P_0}{P_1} \right)^{\frac{\gamma-1}{\gamma}} - 1 \right] \\[2mm]
\Delta T_{1-2} = \delta \left[W_1^2 - W_2^2 \right] \Bigg\} \qquad \text{See figure (2)}$$

Thus K_L can be expressed according to equation (3) as follows:

$$K_L = \frac{W_1^2 - W_2^2}{W_1^2} - \frac{\left(\frac{P_2}{P_1} \right)^{\frac{\gamma-1}{\gamma}} - 1}{\left(\frac{P_0}{P_1} \right)^{\frac{\gamma-1}{\gamma}} - 1}$$

or with equation (2)

$$K_L = \frac{W_1^2 - W_2^2}{W_1^2} - K_\rho$$

Hence:

$$\frac{W_2}{W_1} = \sqrt{1 - K_L - K_\rho} \quad \text{--------------- (7)}$$

The term $\frac{\rho_2}{\rho_1}$ in equation (6) can be replaced by

$$\frac{\left(1 + \frac{\gamma-1}{2} M_1^2 K_\rho \right)^{\frac{\gamma}{\gamma-1}}}{1 + \frac{\gamma-1}{2} M_1^2 (K_\rho + K_L)}$$

which can be shown as follows:

$$\frac{\rho_2}{\rho_1} = \frac{P_2}{P_1} \cdot \frac{T_1}{T_2} \qquad\qquad T_2 = T_1 + b\left(W_1^2 - W_2^2\right) \qquad \text{see figure (2)}$$

with equation (7): $T_2 = T_1 + bW_1^2(K_L + K_P)$ Since $bW_1^2 = \frac{\gamma-1}{2} T_1 M_1^2$, see figure (2), it follows that $\frac{\rho_2}{\rho_1} = \frac{P_2}{P_1} \frac{1}{1 + \frac{\gamma-1}{2} M_1^2 (K_L + K_P)}$ or with equation (2)

$$\frac{\rho_2}{\rho_1} = \frac{\left(1 + \frac{\gamma-1}{2} M_1^2 K_P\right)^{\frac{\gamma}{\gamma-1}}}{1 + \frac{\gamma-1}{2} M_1^2 (K_P + K_L)} \qquad \text{\_ \_ \_ \_ \_ \_ \_ \_ \_} \quad (8)$$

Thus the foregoing substitutions result in equation (5).

From equation (5) it follows, that the pressure coefficient can be determined, if the intake conditions, the turning angle, and the loss coefficient are given, or the loss coefficient can be determined if the intake conditions, the turning angle and the pressure coefficient are given. An agreement between the value of the loss coefficient obtained by this method with the value derived from the total pressure measurements is an indication of the accuracy of measurement achieved. However, for given intake conditions, loss and pressure coefficients, the turning angle cannot be uniquely determined by equation (5) unless the approximate value of $\Delta\beta$ is known since $\sin(\beta_1 + \Delta\beta) = \sin(180 - \beta_1 - \Delta\beta)$ resulting in two possible solutions for $\Delta\beta$.

The velocity triangle corresponding to the cascade flow can be constructed for given intake conditions, turning angle and pressure or loss coefficient as follows: First either the loss or pressure coefficient is determined by equation (5). Then the value of W_2 can be determined by equation (7). By β_1 and $\Delta\beta$, the exit angle β_2 is determined. Since in addition, the intake flow conditions to the cascade are known, the complete cascade velocity triangle is determined.

II. Relationships Between the Cascade Flow Coefficients

And The Obtainable Pressure Ratio and Efficiency of a

Compressor Element*

In line with the main purpose of the two-dimensional cascade investigations, which is to obtain information for the design of compressors, the relationships between adiabatic compressor efficiency, compressor pressure ratio, and the cascade flow coefficients will be given in the following.

In figure 4, a velocity triangle for a compressor element is given. With the definition of pressure coefficient (equation 2) it follows that:

$$\frac{P_2}{P_1} = \left(K_{p,m} \frac{b W_1^2}{T_1} + 1 \right)^{\frac{\gamma}{\gamma-1}} = \left(K_{p,m} \frac{\gamma-1}{2} M_{1,m}^2 + 1 \right)^{\frac{\gamma}{\gamma-1}}$$

and

$$\frac{P_3}{P_2} = \left(K_{p,f} \frac{b C_1^2}{T_1} + 1 \right)^{\frac{\gamma-1}{\gamma}} = \left(K_{p,f} \frac{\gamma-1}{2} M_{1,f}^2 + 1 \right)^{\frac{\gamma}{\gamma-1}}$$

Thus

$$\frac{P_3}{P_1} = \left[\left(K_{p,m} \frac{\gamma-1}{2} M_{1,m}^2 + 1 \right) \left(K_{p,f} \frac{\gamma-1}{2} M_{1,f}^2 + 1 \right) \right]^{\frac{\gamma}{\gamma-1}} \; - - - - - - (9)$$

When C_3 equals C_1 and $b C_1^2$ is small in comparison to T_1 which will be justified for subsonic axial compressor stages, the total pressure ratio $\frac{P_{t,3}}{P_{t,1}}$ equals with good approximation $\frac{P_3}{P_1}$.

The definition of adiabatic compressor efficiency is:

$$\eta_{ad} = \frac{\Delta T_i}{\Delta T}$$

where

$$\Delta T = 2 b U \Delta C_U = b \left(W_1^2 - W_2^2 + C_2^2 - C_1^2 \right)$$

* A compressor element consists of a rotor and stator row with infinitesimal blade length, whereby no wall influence upon the flow is assumed. It will be assumed that the distance between rotor and stator rows is sufficiently large to justify the assumption of uniform and stationary flow at the entrance to the blade rows.

and
$$\Delta T_i = \Delta T - K_{L,m} \, b W_1^{\,2} - K_{L,f} \, b C_2^{\,2}$$

Thus:

$$\eta_{ad} = 1 - \frac{K_{L,m} W_1^{\,2} + K_{L,f} \, C_2^{\,2}}{W_1^{\,2} - W_2^{\,2} + C_2^{\,2} - C_1^{\,2}} \quad -------- \quad (10)$$

Equation (10) determines the adiabatic efficiency in terms of the cascade loss coefficients and the intake and exit velocities of the rotor and stator cascade. By using equation (7), the expression for the adiabatic efficiency can be transformed as follows:

$$\eta_{ad} = 1 - \frac{K_{L,m} W_1^{\,2} + K_{L,f} \, C_2^{\,2}}{W_1^{\,2}(K_{L,m} + K_{P,m}) + C_2^{\,2}(K_{L,f} + K_{P,f})}$$

or,

$$\eta_{ad} = \frac{W_1^{\,2} K_{P,m} + C_2^{\,2} K_{P,f}}{W_1^{\,2}(K_{L,m} + K_{P,m}) + C_2^{\,2}(K_{L,f} + K_{P,f})} \quad -------- \quad (11)$$

Equation (11) determines the adiabatic efficiency of a compressor element in terms of the loss and pressure coefficients of rotor and stator and of the relative intake velocities to the rotor and stator.

For a compressor element with the reaction degree* zero, that is

$$W_1 = W_2 \qquad and \qquad K_{L,m} = -K_{P,m}$$

which follows from equation (7) for $W_1 = W_2$, equation (11) becomes

$$\eta_{ad} = \frac{K_{P,f}}{K_{P,f} + K_{L,f}} - \frac{K_{L,m} \dfrac{W_1^{\,2}}{C_2^{\,2}}}{K_{P,f} + K_{L,f}} \quad -------- \quad (12)$$

* Reaction degree R is defined as:

$$R = \frac{\frac{1}{2}(W_1^{\,2} - W_2^{\,2})}{U \cdot \Delta C_U} = \frac{W_1^{\,2} - W_2^{\,2}}{W_1^{\,2} - W_2^{\,2} + C_2^{\,2} - C_1^{\,2}}$$

For a compressor element with 50% reaction that is, $W_2 = C_1$ and $W_1 = C_2$ and, hence $K_{p,f} = K_{p,m}$ and $K_{L,f} = K_{L,f}$, equation (11) becomes:

$$\eta_{ad} = \frac{K_p}{K_L + K_p} \quad ------------ \quad (13)$$

III. Quality Factors for Cascades

As was shown in Chapter (1), the cascade flow is completely determined by the pressure or loss coefficient, the turning angle, and the intake conditions.

However, in general, it is not possible to draw an immediate conclusion from these coefficients as to the quality of a cascade. Only for the case that the same velocity triangle is basic for a comparison of different cascades is the value of the loss coefficient indicative of the best cascade for this particular velocity triangle. In most cases the quality of a cascade has to be characterised without reference to a prescribed velocity triangle. For this purpose cascade quality factors were defined. Two of the most commonly used are:

a) The so-called cascade efficiency, $\eta_c = \dfrac{K_p}{K_p + K_L}$

(14)

b) The cascade quality factor, $q = \dfrac{K_p}{K_L}$

Both coefficients have their optimum value for the same flow conditions since $\eta_c = \dfrac{q}{q+1}$ and, hence η_c is a maximum if q is a maximum. The expression for η_c in (14) corresponds to the efficiency definition of straight diffusors. With equation (7) η_c can be transformed into $\eta_c = \dfrac{K_p \, W_1^2/2}{W_1^2/2 - W_2^2/2}$; for the limiting case of small Mach numbers this expression represents the ratio of actual static pressure increase through the cascade to the theoretically possible static pressure increase. These coefficients are very practical if, for example, optimum blade configurations for a 50% reaction compressor element are to be determined. This follows from the fact that the expression

for the adiabatic efficiency of a 50% reaction compressor element, equation (13), is identical with the expression of the so-called cascade efficiency. However, these coefficients lose their meaning when the cascade approaches the impulse type, that is, when the function of the cascade is to deflect the flow rather than to produce a pressure rise.

This shows that it is very difficult to define a cascade quality factor which is universally meaningfull. The definition of such a quality factor should take into account both flow deflection and pressure increase through the cascade.

IV. Test Evaluation Methods for Determination

Of the Loss Coefficient

Three methods of determining the losses will be discussed.

   1)   The evaluation of the losses, or the loss coefficient, based upon the stagnation pressure measurements before and behind the cascade.

   2)   The evaluation of the losses, or the loss coefficient, based upon the angles β_1 and β_2 , see figure (1), and upon the static pressure before and behind the cascade.

   3)   The evaluation of the losses based upon the force components acting upon the middle blade.

The Determination of the Loss Coefficient Based upon the Stagnation Pressure Behind the Cascade:

   The first expression for the loss coefficient given in equation (3) can be transformed with good approximation* as follows (see figure (2):

$$K_L = \frac{T_{t,1} - T_{t,2}}{T_{t,1} - T_1}$$

* Valid as long as the pressure line in a T-S diagram corresponding to the static and stagnation pressures behind the cascade are nearly parallel. This condition will be fulfilled for high subsonic and low supersonic speeds. For higher supersonic speeds the exact expression in equation (3) must be applied.

Thus

$$K_L = 1 - \frac{\left(\frac{p_{t,2}}{p_1}\right)^{\frac{\gamma-1}{\gamma}} - 1}{\left(\frac{p_{t,1}}{p_1}\right)^{\frac{\gamma-1}{\gamma}} - 1} = \frac{1 - \left(\frac{p_{t,2}}{p_{t,1}}\right)^{\frac{\gamma-1}{\gamma}}}{1 - \left(\frac{p_1}{p_{t,1}}\right)^{\frac{\gamma-1}{\gamma}}} \qquad \text{---------} \quad (15)$$

or in good approximation, since $p_{t,2}/p_{t,1}$ will be very close to unity:

$$K_L = \frac{\gamma-1}{\gamma} \frac{p_{t,1} - p_{t,2}}{p_{t,1}} \frac{1}{1 - \left(\frac{p_1}{p_{t,1}}\right)^{\frac{\gamma-1}{\gamma}}}$$

or

$$K_L = \frac{\gamma-1}{\gamma} \frac{\Delta p_{t,1-2}}{p_{t,1}} \left(1 + \frac{2}{(\gamma-1) M_1^2}\right) \qquad \left.\begin{array}{c} \\ \\ \\ \\ \end{array}\right\} \quad \text{-----} \quad (16)$$

The Determination of the Loss Coefficient Based Upon Static Pressure Ratios, The Turning Angle, and the Inlet Condition: Equation (5) can be written by using equation (2) as follows:

$$1 = \frac{\sin(\beta_1 + \Delta\beta)}{\sin\beta_1} \frac{p_2}{p_1 M_1^2} \frac{2}{\gamma-1} \frac{\sqrt{1 - K_L - K_p}}{\frac{2}{(\gamma-1)M_1^2} + K_L + K_p} \qquad \text{------} \quad (17)$$

or, in abbreviated form, equation (17) can be written as follows: (18)

$$1 = B \frac{\sqrt{1 - K_L - K_p}}{D + K_L + K_p}$$

This equation solved for K_L explicitly, results in the following equation:

$$K_L = B\left[\sqrt{1 + D + \frac{B^2}{4}}\right] - \left(\frac{B^2}{2} + D + K_p\right) \qquad \text{-------} \quad (18\text{ a})$$

or

$$K_L = \frac{\sin(\beta_1 + \Delta\beta) \frac{p_2}{p_1}}{\sin\beta_1 \frac{\gamma-1}{2} M_1^2}\left(\sqrt{1 + \frac{1}{\frac{\gamma-1}{2}M_1^2} + \frac{1}{4}\left(\frac{\sin(\beta_1 + \Delta\beta)\frac{p_2}{p_1}}{\sin\beta_1 \frac{\gamma-1}{2}M_1^2}\right)^2} - \frac{1}{2}\left(\frac{\sin(\beta_1 + \Delta\beta)\frac{p_2}{p_1}}{\sin\beta_1 \frac{\gamma-1}{2}M_1^2}\right)\right) -$$

$$- \frac{\left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}}}{\frac{\gamma-1}{2}M_1^2} \qquad \text{-----------------} \quad (18\text{b})$$

Equation (18) expresses the loss coefficient explicitly as function of the measured static pressure ratio p_2/p_1, the intake conditions, β_1 and M_1, and the turning angle $\Delta\beta$.

A study of equation (18) shows that the loss coefficient, K_L, is expressed as the difference of two large magnitudes. Hence in measuring the magnitudes a high accuracy is required in order to determine K_L accurately. If the loss coefficient can be determined more accurately by measuring the total pressure drop through the cascade, then the relationship expressed by equations (17) or (18) can be used for correcting either the turning angle or the pressure coefficient.

The Determination of the Loss and Pressure Coefficients Based Upon Force Measurements:

For given flow conditions upstream of the cascade, the turning angle, the pressure and loss coefficients can be determined if the force acting upon one middle blade is known in amount and direction.

From the momentum law it follows that:

$$F_U = m\left(W_{1U} - W_{2U}\right) \qquad (19)$$

and

$$F_{ax} = sh\left(p_2 - p_1\right) - m\left(W_{ax,1} - W_{ax,2}\right) \qquad (20)$$

From the continuity condition it follows that

$$\rho_1 W_{ax,1} = \rho_2 W_{ax,2} \qquad (21)$$

Combining equations (20) and (21) results in:

$$\frac{p_2}{p_1} = \frac{F_{ax}\,\sin\beta_1}{s\cdot h\cdot p_1}\,W_1\left(1 - \frac{\rho_1}{\rho_2}\right) + 1 \qquad (22)$$

where on the right hand side only the ratio p_1/p_2 is unknown. The reciprocal of p_1/p_2 can be expressed as follows:

$$\frac{p_2}{p_1} = \frac{p_2\, T_1}{p_1\, T_2} = \frac{p_2\, T_1}{p_1\left(T_1 + b\left(W_1^2 - W_2^2\right)\right)} \quad - - - - - - - - \quad (23)$$

By using equations (19) and (21) it is possible to express p_2/p_1 , by p_2/p_1 and T_1 , as follows:

$$W_{ax,2} = \frac{p_1}{p_2}\, W_1 \sin\beta_1, \qquad\qquad W_{2u} = -\frac{F_u}{m} + W_1 \cos\beta_1$$

Thus

$$W_2^2 = W_1^2\left[\left(\frac{p_1}{p_2}\sin\beta_1\right)^2 + \cos^2\beta_1\right] + \left(\frac{F_u}{m}\right)^2 - 2\frac{F_u}{m}\, W_1 \cos\beta_1$$

Substitution of W_2^2 into equation (23) results in

$$\frac{p_2}{p_1} = \frac{\dfrac{p_2}{p_1}}{1 + \dfrac{b}{T_1}\left[W_1^2 \sin^2\beta_1 \left(1 - \left(\frac{p_1}{p_2}\right)^2\right) - \left(\frac{F_u}{m}\right)^2 + \frac{2F_u}{m}\, W_1 \cos\beta_1\right]}$$

Solving for p_2/p_1 explicitly, results in:

$$\frac{p_2}{p_1} = \frac{\dfrac{p_2}{p_1} + \sqrt{\left(\frac{p_2}{p_1}\right)^2 + 4\left[1 + \frac{b}{T_1}\left(W_1^2 \sin^2\beta_1 - \left(\frac{F_u}{m}\right)^2 + \frac{2F_u}{m}W_1 \cos\beta_1\right)\right]\dfrac{b\,W_1^2 \sin^2\beta_1}{T_1}}}{2\left[1 + \frac{b}{T_1}\left(W_1^2 \sin^2\beta_1 - \left(\frac{F_u}{m}\right)^2 + \frac{2F_u}{m}W_1 \cos\beta_1\right)\right]} \tag{24}$$

where

$$T_1 = T_o\left(\frac{p_1}{p_2}\right)^{\frac{\gamma-1}{\gamma}}$$

and

$$m = s \cdot h \cdot W_1\, p_1 = p_o\frac{T_o}{T_1}\frac{p_1}{p_o}\, s \cdot h \cdot W_1$$

or
$$m = P_0 \left(\frac{p_1}{p_0}\right)^{\frac{1}{\gamma}} s \cdot h \cdot W_1$$

Substituting the expression for P_2/p_1 of equation (24) into equation (22) determines the pressure ratio P_2/p_1 and hence p_2/p_1 as functions of the blade force and of the flow conditions upstream of the cascade. Furthermore, equation (19) and (21) determine the downstream velocity and hence the turning angle by substituting the solution for p_2/p_1 into equation (21). Thus, the downstream flow conditions are completely determined by the amount and direction of the blade force and by the up stream flow conditions. Therefore, the pressure and loss coefficients can be determined by equations (2) and (18) respectively.

The above consideration shows that it is possible to relate the blade force with the flow conditions far behind the cascade where the wakes are dispersed. These relationships hold provided that the momentum transport of the flow behind the cascade is constant, which means that no interference between the tunnel walls and the cascade flow exists. However in measuring the amount and direction of the blade force a very high accuracy is required since the loss coefficient is to be determined by using equation (18), If by measuring other magnitudes the cascade loss coefficient can be determined more accurately, then the relationships derived in this chapter could be used for an indirect determination of the blade force. For this purpose it would be practical to transform equations (19) and (20) as follows:

$$F_U = m W_1 \left[\cos\beta_1 - \cot(\beta_1 + \Delta\beta) \sin\beta_1 \, \frac{P_1}{P_2} \right] \tag{25}$$

and

$$F_{ax} = s \cdot h \cdot p_1 \left(\frac{P_2}{P_1} - 1\right) - m W_1 \sin\beta_1 \left(1 - \frac{p_1}{p_0}\right) \tag{26}$$

or

$$F_{ax} = s \cdot h \cdot p_1 \left\{ \left[K_p \left\{ \left(\frac{p_1}{p_0}\right)^{\frac{\gamma-1}{\gamma}} - 1 \right\} + 1 \right]^{\frac{\gamma}{\gamma-1}} - 1 \right\} - m W_1 \sin\beta_1 \left(1 - \frac{p_1}{p_2}\right) \tag{27}$$

The ratio p_1/p_2 in these equations can be replaced by the following expression which is derived from equations (5) and (8):

$$\frac{\rho_1}{\rho_2} = \frac{\sin(\beta_1 + \Delta\beta)}{\sin\beta_1} \sqrt{1 - K_L - K_\rho}' \quad \text{------------ (28)}$$

The equations 25, 27, and 28 give the relationships between the blade force components, the intake flow conditions, the turning angle, the loss and pressure coefficient. They may be used for expressing a cascade drag and lift coefficient defined for compressible flow in terms of flow deflection, pressure and loss coefficient.

NOMENCLATURE

A = Cross sectional area of the flow

b = $\dfrac{\text{Total temperature} - \text{Static temperature}}{(\text{Velocity})^2} - \dfrac{1}{2Jgc_p}$

C = Absolute velocity

c_p = Specific heat at constant pressure

c_v = Specific heat at constant volume

F = Force acting upon a blade

g = Acceleration due to gravity

h = Height of a blade

J = Mechanical equivalent of heat

K_L = Loss coefficient

K_p = Pressure coefficient

m = Mass flow per sec. through a channel formed by two adjacent blades

M = Local Mach number

p = Pressure

q = Cascade quality factor

s = Distance between two blades

S = Entropy

T = Absolute temperature

U = Circumferential velocity of a blade row

W = Velocity relative to the cascade

β = Angle between flow direction and cascade axis

γ = $\dfrac{c_p}{c_v}$

Δ = Finite change in magnitude

η = Efficiency

ρ = Mass density

SUBSCRIPTS

i = Gas conditions for the case that no entropy increase occurs through the cascade, and the pressure increase through the cascade is the same as in the actual case.

0 = Ambient atmospheric conditions

1 = Intake conditions of the flow to the blade row (for a cascade, conditions at station (1), figure (1).

2 = Condition of the flow behind a cascade (station (2), figure (1).

3 = Conditions of the flow behind a compressor stage.

ad = Adiabatic

ax = Axial component (see figure (1).

C = Cascade

f = Stationary cascade

m = Moving cascade

t = Total conditions of a gas

U = Tangential components

SUPERSCRIPTS

* = Critical conditions
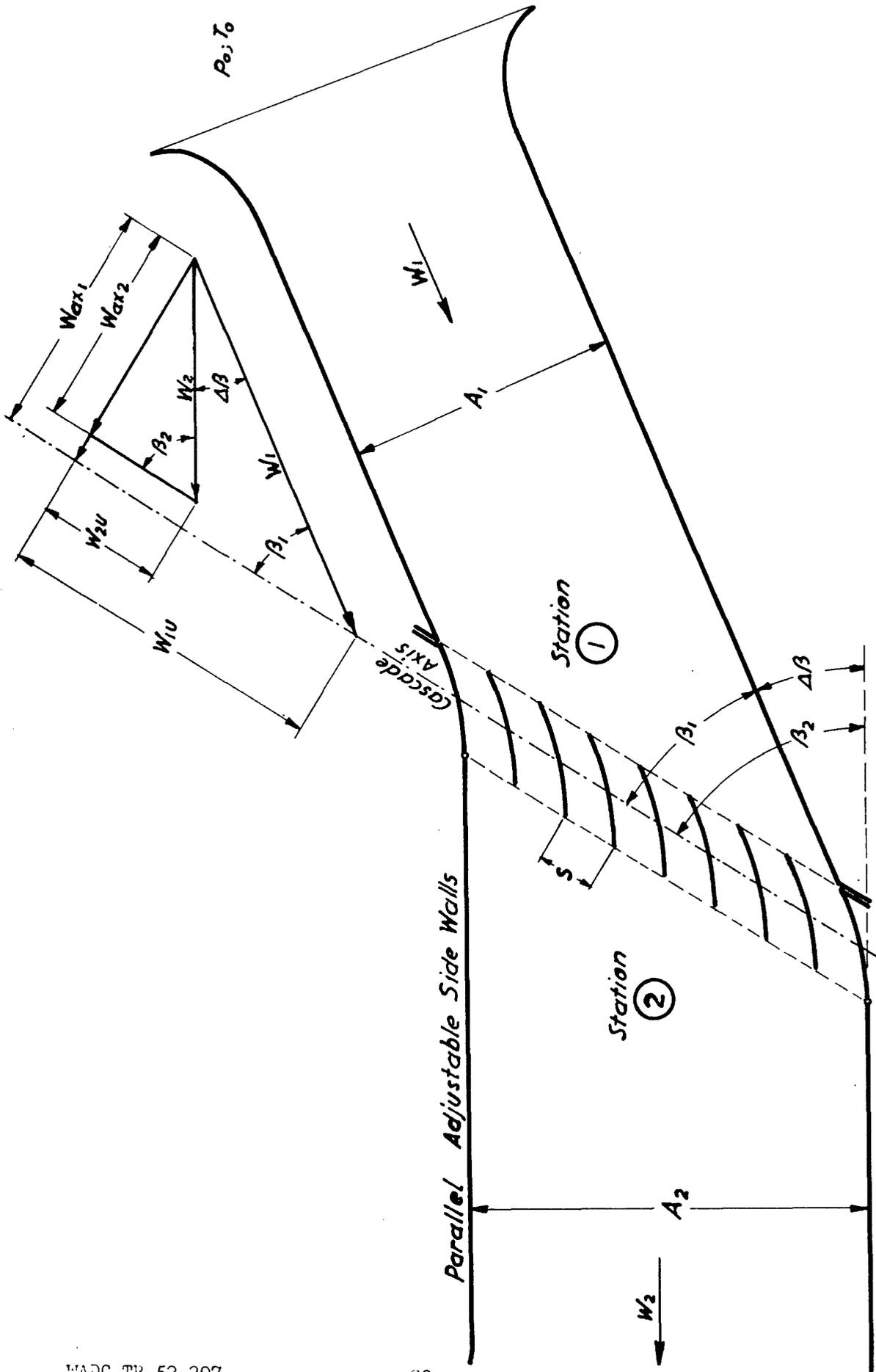
Figure 1. Scheme of Cascade Tunnel

$$p_o = p_{t,1}$$

$$p_{t,2}$$

$$T_o = T_{t,1}$$

$$T_{i,t,2}$$

$$T_{t,1} - T_1 \begin{cases} = \dfrac{1}{2J \cdot g \cdot c_p} \cdot W_1^2 = b \cdot w_1^2 \\[2mm] = T_1\left[\left(\dfrac{p_o}{p_1}\right)^{\frac{\gamma-1}{\gamma}} - 1\right] \\[2mm] = T_1 \dfrac{\gamma-1}{2} M_1^2 = T_1 \cdot 0.2\, M_1^2 \\[2mm] = T_o\left[1 - \left(\dfrac{p_1}{p_o}\right)^{\frac{\gamma-1}{\gamma}}\right] \\[2mm] = T_o \dfrac{\frac{\gamma-1}{2} M_1^2}{1 - \frac{\gamma-1}{2} M_1^2} \end{cases}$$

$$b w_2^2 = T_o\left[1 - \left(\dfrac{p_2}{p_{t,2}}\right)^{\frac{\gamma-1}{\gamma}}\right]$$

$$p_2$$

$$T_2$$

$$\Delta T_{1-2} - \Delta T_{i,1-2} \sim \dfrac{\Delta S_{1-2}\, T_2}{c_p}$$

$$\Delta T_{1-2} = b\left(w_1^2 - w_2^2\right)$$
$$= T_o\left[\left(\dfrac{p_2}{p_{t,2}}\right)^{\frac{\gamma-1}{\gamma}} - \left(\dfrac{p_1}{p_o}\right)^{\frac{\gamma-1}{\gamma}}\right]$$

$$\Delta T_{i,1-2} \begin{cases} = T_1\left[\left(\dfrac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} - 1\right] \\[2mm] = T_o\left[\left(\dfrac{p_2}{p_o}\right)^{\frac{\gamma-1}{\gamma}} - \left(\dfrac{p_1}{p_o}\right)^{\frac{\gamma-1}{\gamma}}\right] \end{cases}$$

$$p_1$$

$$S_1 \qquad S_2$$

$$T_1 = T_o\left(\dfrac{p_1}{p_o}\right)^{\frac{\gamma-1}{\gamma}}$$
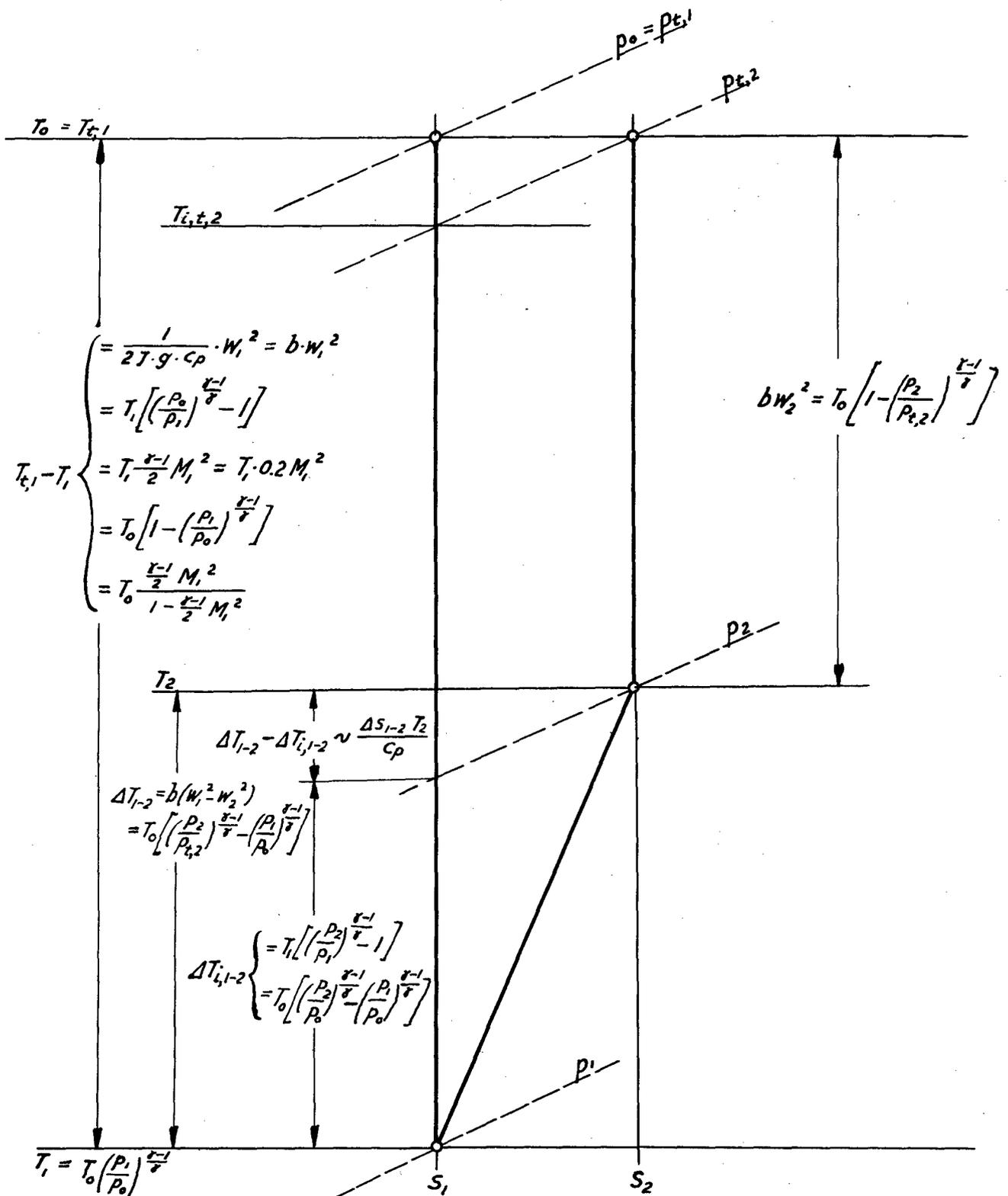
Figure 2. Cascade Process in T-S Diagram.

I   Parameters determining the profile form and the cascade geometry.

II   Flow parameters : Reynolds Number , turbulence

III Plots of the turning angle, pressure and loss coefficient versus the intake angle. Curve parameter is the intake Machnumber M_1
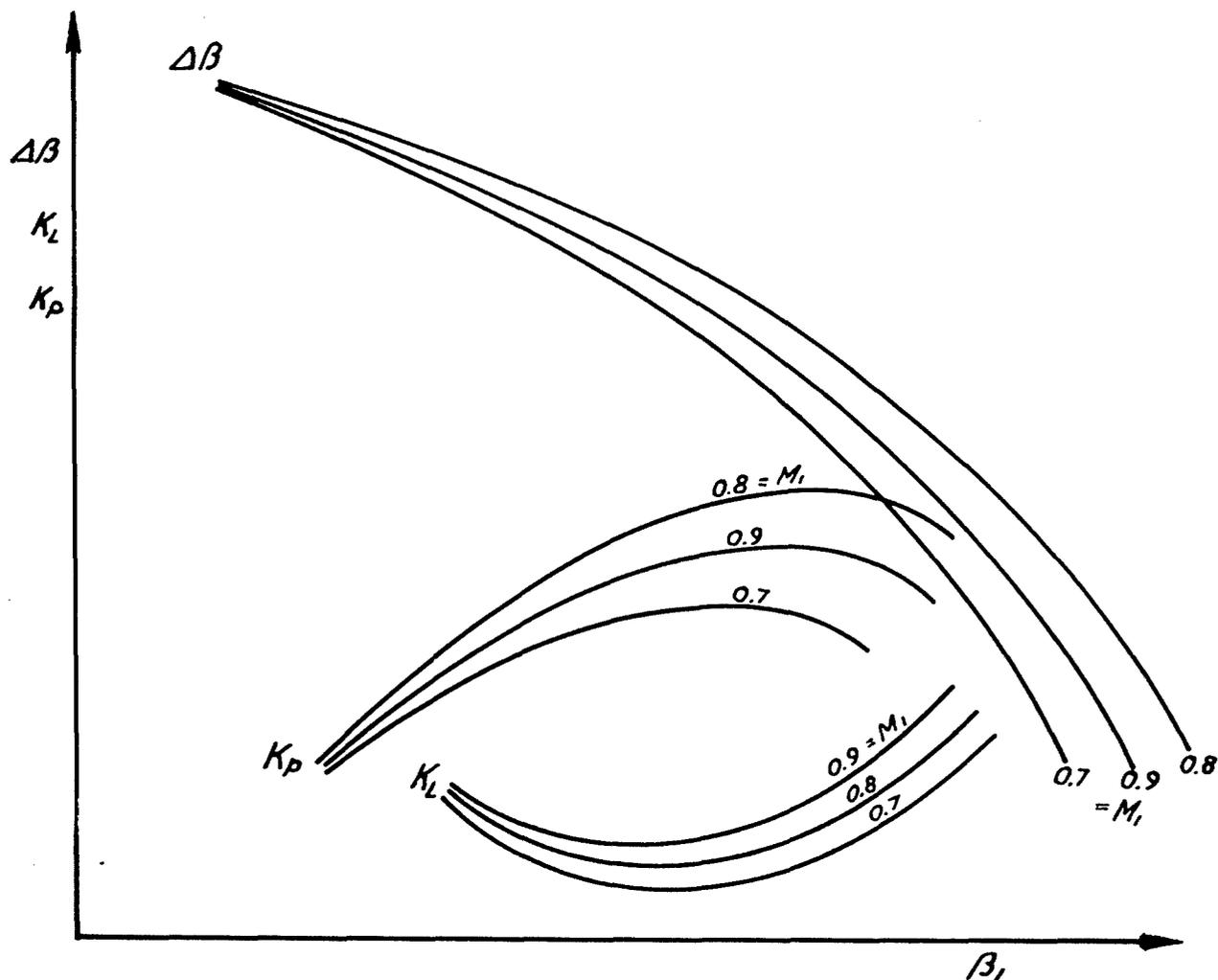

Figure 3. Scheme for Presentation of Cascade Properties.
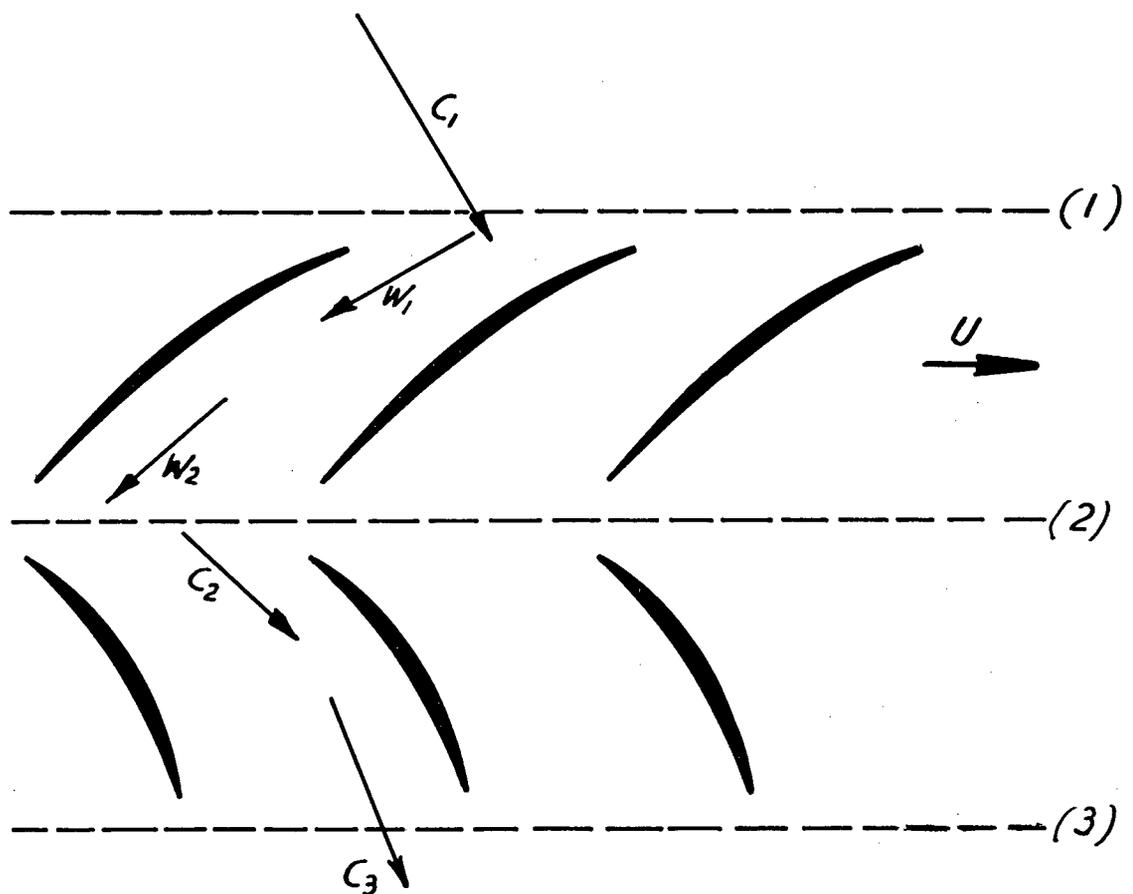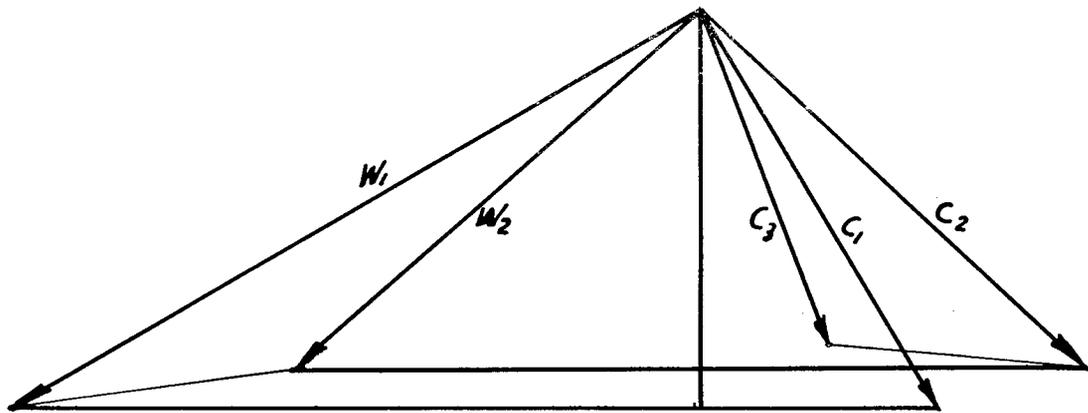
Figure 4. Velocity Triangle for a Compressor Element.