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by

R. D. MINDLIN and H. DERESIEWICZ

Office of Naval Research Project NR-064-388
Contract Nonr-266(09)
Technical Report No. 10
CU-11-53-ONR-266(09)-CE
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Introduction

The range of applicability of the one-dimensional theory of flexural vibrations of beams was extended to higher frequencies by Timoshenko when he took into account the effect of transverse shear deformation. He arrived at the free-vibration equations (1)*

$$EI\frac{\partial^2\psi}{\partial x^2} + k\left(\frac{\partial y}{\partial x} - \psi\right)AG - \frac{I\gamma}{g}\frac{\partial^2\psi}{\partial t^2} = 0$$

$$\frac{\gamma A}{g}\frac{\partial^2 y}{\partial t^2} - k\left(\frac{\partial^2 y}{\partial x^2} - \frac{\partial\psi}{\partial x}\right)AG = 0 \qquad [1]$$

governing the transverse deflection, y , and the slope, ψ , of the deflection curve when the shear is neglected. The coefficient k is defined as the ratio of the average shear stress on a section to the product of the shear modulus and the angle of shear at the neutral axis. This ratio depends upon the distribution of shear stress on the section and, hence, k depends upon the shape of the section, as Timoshenko observed. However, the distribution of shear stress on a section depends also on the mode of motion of the beam. For example, for low modes of motion of a slender beam, the shear stress has a maximum at the neutral axis, while, for very high modes, the shear stress has a minimum at the same place. Thus, k depends both on the shape of the section and the frequency of vibration.

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* Numbers in parenthesis refer to Bibliography at the end of the paper.

In the solution of Equations [1], the simplest interpretation of k is that it is a constant. For a beam of given cross-sectional shape, then, k can have the correct value for only one frequency. In the past, several calculations of k have been made, for various cross-sectional shapes, on the basis of statical considerations, that is, at zero frequency. These values are satisfactory for the low modes of motion of slender beams, where, in fact, the influence of transverse shear deformation is small in comparison with its influence at high frequencies.

As the frequency is increased, a point is reached at which a drastic change occurs in the spectrum. This is the frequency, ω', of the first thickness-shear mode of a beam of infinite length, i.e., the lowest frequency at which the infinite beam can vibrate with no transverse deflection, the displacement being entirely parallel to the axis of the beam.

The reason for the change in character of the frequency spectrum is that, at the frequencies of the first thickness-shear mode and its overtones, there is strong coupling between the flexural and thickness-shear modes of motion. Examples of this phenomenon have been described in detail in previous papers (2, 3) for special cases of crystal plates for which the equations of motion are the same as those for a Timoshenko beam.

It is desirable, then, that the thickness-shear frequency calculated from Equations [1] should be the same as that calculated from the equations to which [1] are an approximation, namely, the three-dimensional equations of small vibrations of an elastic body. In this connection it must be recognized that, whereas Timoshenko interpreted ψ as the slope of the deflection curve when the shear deformation is neglected, the product of ψ and the depth of the beam may be interpreted as the maximum axial displacement of a transverse section. Thus, ω' is calculated from [1] by setting $y = 0$ and ψ proportional to $exp(i\omega' t)$, following which ω' is set equal to the corresponding frequency calculated from the three-dimensional equations. Since ω' depends upon k, the result is a formula for k.

It has been shown (4) that, for a rectangular section, this procedure leads to $k = \pi^2/12 \approx 0.822$. Comparison with experiments (2, 3) shows that, when k is calculated in this manner, Timoshenko's equations give good results for both low and high frequencies.

In the present paper, k is calculated in the same manner for a variety of cross-sectional shapes. To do this, it is necessary to solve the three-dimensional equations of elasticity for the appropriate frequency and equate it with the value of ω' obtained from Equations [1]. In the case of motion parallel to the axis of the bar, the three-dimensional equations and boundary conditions reduce to equations governing a familiar hydrodynamical problem for which many solutions are known. The determination of k is thus reduced to an interpretation of these solutions and some additional computations. Results are given for the following sections: circle, ellipse, orthogonal parabolas and a variety of ovaloids. The values of k computed for these sections all lie within about 10% of that for the rectangular section.

Thickness-Shear Motion: Timoshenko Theory

If y is set equal to zero in Equations [1], the second of the equations gives $\partial\psi/\partial x = 0$, so that the first of [1] reduces to

$$\frac{d^2\psi}{dt^2} + k\left(\frac{c}{r}\right)^2\psi = 0 \qquad [2]$$

where $c = (Gg/\gamma)^{1/2}$, i.e., the velocity of shear waves in an infinite, isotropic, elastic medium, and $r = (I/A)^{1/2}$, i.e., the radius of gyration of the cross-section. Hence, the frequency of pure thickness-shear vibration of a Timoshenko beam is

$$\omega' = c k^{1/2}/r \qquad [3]$$

This is the frequency which is to be equated to the one calculated from the solution of the three-dimensional equations for each section.

Thickness-Shear Motion: Three-Dimensional Theory

Let the neutral axis of the beam be the z-axis of a rectangular coordinate system and let u , v and w be components of displacement in the x , y and z directions, respectively. For pure thickness-shear motion,

$$u = v = 0$$
$$w = \zeta(x, y) e^{i\omega t} \qquad [4]$$

Then the three-dimensional equations of motion (5) reduce to

$$\nabla^2 \zeta + \gamma^2 \zeta = 0 \qquad [5]$$

where $\gamma = \omega/c$ and $\nabla^2 = \partial^2/\partial x^2 + \partial^2/\partial y^2$. The condition that the traction vanish on the cylindrical surface of the beam reduces to

$$\partial \zeta/\partial n = 0 \qquad [6]$$

on the boundary, where n is the normal to the boundary.

The differential equation [5] and the boundary condition [6] are the same as those governing the small oscillations of a fluid in a basin of uniform depth (6) and the small vibrations of a gas in a rigid cylindrical container (7). Solutions are available for a variety of boundaries.

Rectangle

In the case of a rectangular section, the frequency is independent of the width. If the depth is $2a$, the frequency is (6)

$$\omega = \pi c/2a \qquad [7]$$

Equating [7] and [3], and noting that $r^2 = a^2/3$, the shear constant, k , is found to be $\pi^2/12$, or, approximately, 0.822.

Circle

For a circular section of radius a , the lowest antisymmetric mode has a frequency (6)

$$\omega = 1.841\, c/a \qquad [8]$$

The radius of gyration of a circle about a diameter is $a/2$. Hence, equating [8] and [3], $k = 0.847$.

Ellipse

The elliptic section was treated by Jeffreys (8), who computed frequencies for two values of the eccentricity

$$e = (1 - b^2/a^2)^{1/2} \qquad [9]$$

where a and b are the semi-major and semi-minor axes, respectively.

Additional values were computed and are listed in the columns headed $\omega a/c$ and $\omega b/c$ in Table I. The corresponding values of ω were then equated to that in Equation [3] to obtain the values of k listed in Table I.

It is interesting to notice that the frequency ($1.886\ c/a$) of the first antisymmetric mode of a very narrow ellipse, about its minor axis, is greater than that ($1.571\ c/a$) of a rectangle of depth equal to the major diameter of the ellipse. Jeffreys observed that this is due to the concentration of motion near the center of the ellipse. On the other hand, the corresponding frequency for a very wide ellipse appears to approach that of the rectangle. However, the corresponding values of k are not the same because of the difference in radii of gyration.

Parabolas

The hydrodynamic problem for a symmetric section bounded by a pair of orthogonal parabolas was studied by Hidaka (9). Such a section has the property $a/b = 2$, where a and b are the semi-major and semi-minor axes, respectively. For motion antisymmetric with respect to the minor axis, Hidaka found a secular equation which can be reduced to

$$J_{-3/4}(\gamma a/2) = 0 \qquad [10]$$

where $J(x)$ is the Bessel function of the first kind. The lowest root of Equation [10] is (10)

$$\gamma a/2 = 1.0585 \qquad [11]$$

so that

$$\omega = 2.117\ c/a \qquad [12]$$

Equating [12] and [3] and noting that $r^2 = a^2/5$, the result is $k = 0.896$.

This completes the calculations for sections for which exact solutions are available. The results are assembled in Table I and the sections are illustrated in Fig. 1.

Ovaloids

Approximate frequencies, good for narrow sections, may be obtained by neglecting the variation of displacement across the width. Hidaka (9) did this for sections bounded by the curves

$$\frac{x^2}{a^2} + \left(\frac{y^2}{b^2}\right)^{1/2m} = 1 \qquad [13]$$

$$m = 2^{-\mu}, \quad \mu = 0, 1, 2, \cdots$$

A set of these curves, for the case $a = 2b$, is shown in Fig. 2.

Hidaka's results for the lowest mode, antisymmetric about the horizontal axis of Fig. 2, are given in Table II, along with the corresponding values of k. In finding k , the radius of gyration (r), defined by

$$r^2 = \frac{\int_0^a y\, x^2\, dx}{\int_0^a y\, dx} \qquad [14]$$

must be computed. Inserting y from Equation [13] into Equation [14], and making the substitution $x = a \sin\theta$ in the integrals, we find

$$\frac{r^2}{a^2} = 1 - \frac{\int_0^{\pi/2} \cos^{(3+2m)}\theta\, d\theta}{\int_0^{\pi/2} \cos^{(1+2m)}\theta\, d\theta} \qquad [15]$$

By aid of the formula (11)

$$\int_0^{\pi/2} \cos^n x \, dx = \frac{\sqrt{\pi}}{2} \frac{\Gamma\left(\frac{n+1}{2}\right)}{\Gamma\left(\frac{n}{2}+1\right)} , \qquad n > -1$$

and the recursion formula for the Gamma function, Equation [15] reduces to

$$r^2 = a^2/(3 + 2m) \qquad [16]$$

The results in Table II are the same as in Table I for the rectangle ($\mu = \infty$) for any b/a since the assumption of uniform displacement across the width is exact in pure thickness-shear motion. For the ellipse ($\mu = 1$), Table II gives the same value as Table I only for $b/a = 0$ (i.e., $e = 1$), since here, too, there is no variation of displacement across the width. As the section becomes wider, there is some discrepancy, but it is not great. Even for $a = b$ (the circle), Table I gives k = 0.847, while the approximation yields 0.889. The approximation is not intended, of course, to apply to $b > a$. Another comparison may be made for the parabola with $a = 2b$. The value of k in Table I is 0.896, while the approximation gives 0.924. Thus it may be expected that the approximate solution of Equation [5] will give results good to within a few percent for sections at least twice as deep as they are wide.

TABLE I

	Motion antisymmetric about minor axis		Motion antisymmetric about major axis	
	$\omega a/c$	k	$\omega b/c$	k
<u>Rectangle</u>	1.571	.822	1.571	.822
<u>Ellipse</u>				
e = 1.0	1.886*	.889	**	
e = 0.9	1.878	.882	**	
e = 0.8	1.87 *	.87	1.78 *	.79
e = 0.7	1.858	.863	1.814	.823
e = 0.6	1.856	.861	1.823	.831
e = 0.3	1.845	.851	1.837	.844
e = 0 (circle)	1.841	.847	1.841	.847
<u>Orthogonal Parabolas</u>	2.117	.896		

* Jeffreys (8)
** Convergence slow

TABLE II

μ	m	$\omega a/c$	r^2/a^2	k
0	1 *	2.150	1/5	.924
1	1/2 **	1.886	1/4	.889
2	1/4	1.737	2/7	.862
3	1/8	1.655	4/13	.843
4	1/16	1.616	8/25	.836
5	1/32	1.591	16/49	.826
∞	0 ***	1.571	1/3	.822

* parabolas; ** ellipse; *** rectangle.

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It is of some historical interest that both the rotatory inertia correction, usually attributed to Rayleigh ("Theory of Sound," Cambridge, England, first edition, 1877; current edition, reference (7), art. 162), and the transverse shear correction, usually attributed to Timoshenko (Philosophical Magazine, Ser. 6, Vol. 41, 1921, pp. 744-746, and Vol. 43, 1922, pp. 125-131), are given by M. Bresse in his "Cours de Mécanique Appliquée," Mallet-Bachelier, Paris, 1859, p. 126.

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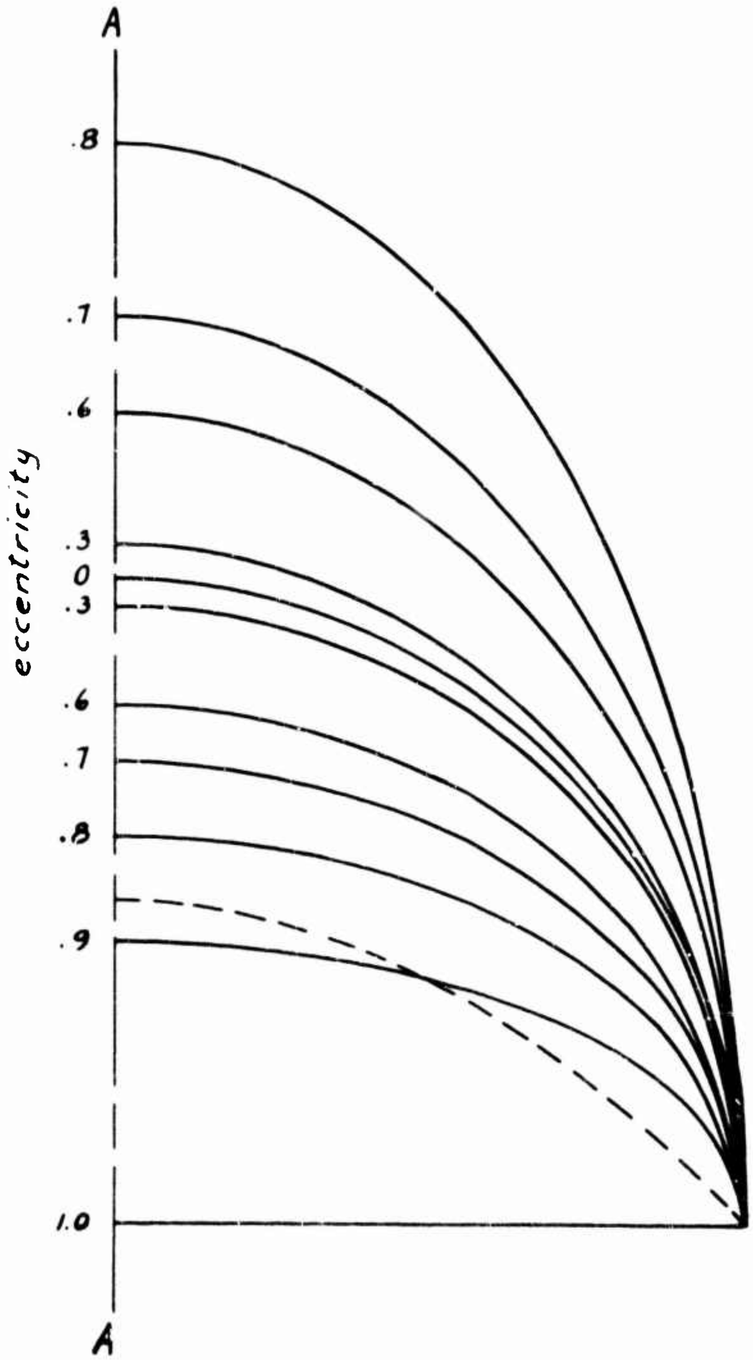

Fig. 1: Elliptic and parabolic sections for which thickness-shear frequency (ω) and Timoshenko's constant (k) are given in Table I. Dashed line is parabolic section. Thickness-shear motion is antisymmetric about axis *A-A*.

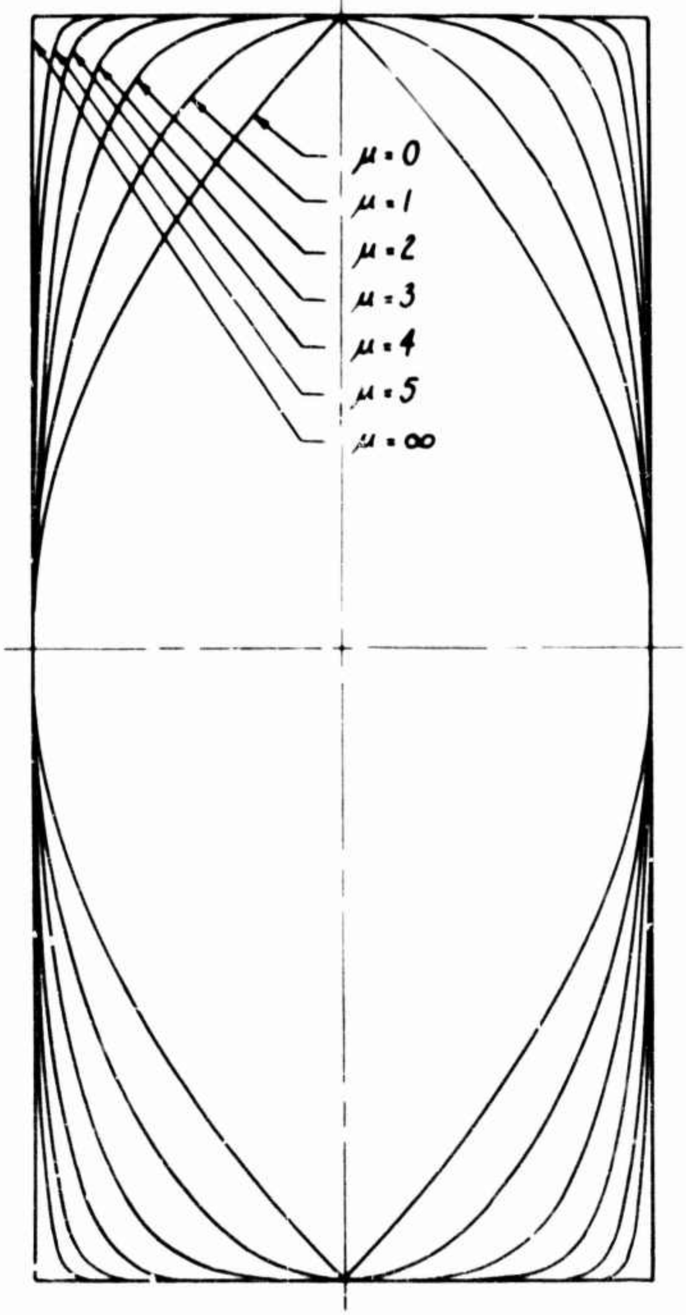

Fig. 2: Ovaloid sections for which approximate values of ω and k are given in Table II.

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