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A.D.C. ANGULAR VELOCITY PICK-OFF FOR USE ON GYROSCOPES

by

F.J. WOODCOCK, A.M.I.E.E., A.F.R.Ae.S.

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A D.C. Angular Velocity Pick-off for Use on Gyroscopes

by

F.J. Woodcock, A.M.I.E.E., A.F.R.Ae.S.

SUMMARY

This note describes an angular velocity pick-off for indicating the rate of change of gimbal displacement of a free gyroscope over a range of ±30°, to within ±7%, with a sensitivity of 0.064 volts per radian per second. The pick-off takes the form of a cylinder 0.9 inches in diameter and 0.4 inches long.

The sensitivity could be increased by about 20% without increasing the volume and it could be increased considerably more by a small increase in volume; e.g., a 100% increase in sensitivity could be obtained with about 30% increase in size. Simple modifications may apparently be incorporated to increase the accuracy and range, or to alter the form of indication so that the sensitivity increases with increasing gimbal displacement.

Development is as yet incomplete, but the construction and method of operation are described and preliminary test results are given and discussed.

As the arrangement of the magnetic circuit appears to be an efficient one for wide-angle transducers, brief consideration is given to the possibility of using the unit as a torsional vibration pick-up or as a torque motor.
# LIST OF CONTENTS

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Introduction</td>
</tr>
<tr>
<td>2</td>
<td>Choice of Type of Pick-off</td>
</tr>
<tr>
<td>3</td>
<td>Construction</td>
</tr>
<tr>
<td>4</td>
<td>Method of Operation</td>
</tr>
<tr>
<td>5</td>
<td>Installation</td>
</tr>
<tr>
<td>6</td>
<td>Test Results</td>
</tr>
<tr>
<td>6.1</td>
<td>Resistance and Inductance Measurements</td>
</tr>
<tr>
<td>6.2</td>
<td>Initial Checks on Sensitivity</td>
</tr>
<tr>
<td>6.3</td>
<td>Tests With Increased Magnetisation</td>
</tr>
<tr>
<td>6.4</td>
<td>Tests With Increased Magnet Thickness</td>
</tr>
<tr>
<td>6.5</td>
<td>Unit Installed to Measure Outer Gimbal Velocity</td>
</tr>
<tr>
<td>7</td>
<td>Discussion of Results</td>
</tr>
<tr>
<td>7.1</td>
<td>Inductance Measurements</td>
</tr>
<tr>
<td>7.2</td>
<td>Natural Frequency, Resonant Frequency and Time Constant</td>
</tr>
<tr>
<td>7.3</td>
<td>Sensitivity</td>
</tr>
<tr>
<td>7.3.1</td>
<td>General Sensitivity Equation</td>
</tr>
<tr>
<td>7.3.2</td>
<td>Application to Present Pick-off</td>
</tr>
<tr>
<td>7.4</td>
<td>Reaction Between Moving Parts</td>
</tr>
<tr>
<td>7.5</td>
<td>Operating Flux Densities</td>
</tr>
<tr>
<td>8</td>
<td>Applications</td>
</tr>
<tr>
<td>8.1</td>
<td>Velocity Pick-off</td>
</tr>
<tr>
<td>8.2</td>
<td>Torsional Vibration Pick-up</td>
</tr>
<tr>
<td>8.3</td>
<td>Torque Motor</td>
</tr>
<tr>
<td>9</td>
<td>Conclusions</td>
</tr>
<tr>
<td>10</td>
<td>Acknowledgement</td>
</tr>
<tr>
<td></td>
<td>Reference</td>
</tr>
<tr>
<td></td>
<td>Advance Distribution</td>
</tr>
<tr>
<td></td>
<td>Detachable Abstract Cards</td>
</tr>
</tbody>
</table>
APPENDIX

Sensitivity and Torque Calculations

LIST OF ILLUSTRATIONS

<table>
<thead>
<tr>
<th>Illustration</th>
<th>Figure</th>
</tr>
</thead>
<tbody>
<tr>
<td>Magnet Assembly</td>
<td>1</td>
</tr>
<tr>
<td>Armature Assembly</td>
<td>2</td>
</tr>
<tr>
<td>Flux Paths Through the Armature</td>
<td>3</td>
</tr>
<tr>
<td>Method of Polarising the Magnet</td>
<td>4</td>
</tr>
<tr>
<td>Pick-off in Position</td>
<td>5</td>
</tr>
<tr>
<td>Equivalent Circuits</td>
<td>6</td>
</tr>
<tr>
<td>Typical Records</td>
<td>7</td>
</tr>
<tr>
<td>Typical Pick-off Output</td>
<td>8</td>
</tr>
<tr>
<td>Relationship Between Velocity and Pick-off Output</td>
<td>9</td>
</tr>
<tr>
<td>Demagnetisation Curve</td>
<td>10</td>
</tr>
<tr>
<td>Equivalent Magnetic Circuits</td>
<td>11</td>
</tr>
<tr>
<td>Sketch Defining Gap Reluctances</td>
<td>12</td>
</tr>
<tr>
<td>Elementary Phase Advance Circuit</td>
<td>13</td>
</tr>
<tr>
<td>Suggested Arrangement for Torque Motor</td>
<td>14</td>
</tr>
</tbody>
</table>
1. Introduction

In some guided missile control systems it is necessary to obtain a heading angle signal and also a rate of change of heading angle. This is the case for example in the current C.T.V. control system for use with command link guidance. The heading angle is obtained from a potentiometer which picks off the gimbal angle of a free gyroscope. This signal is operated upon by phase advance circuits, but certain disadvantages are inherent in this method. The signal/noise ratio of the potentiometer output is low as the output signal necessarily consists of a number of discrete steps and is further distorted by variations in contact resistance. The differentiating action of the phase advance circuits on this output signal produces an intolerably low signal/noise ratio at the amplifier output.

Perhaps the most direct method of overcoming this difficulty is to reduce the noise generated by the pick-off. In this instance the pick-off was a D.C. energised wire-wound potentiometer and appropriate investigations were started.

It appeared that another possible method would be to have two pick-offs on the gyroscope, one to provide a displacement signal and the other to provide a velocity signal. The development of the present velocity pick-off represents the first step in an attempt to find a solution along these lines.

In the following sections, construction, operation and installation are described and some preliminary test results given. These are discussed with the aid of a theoretical analysis of the magnetic circuit and it is also shown that the unit might well form the basis of a torsional vibration pick-up or a wide-angle torque motor.

2. Choice of Type of Pick-off

It was decided initially that the displacement signal would continue to be derived from the existing potentiometer and that the velocity pick-off should fit into the then current gyro frame and gimbal system (Air Blast Gyroscope Type G.W.1 Mark 4) with the minimum of modification.

In addition the pick-off should

(a) need no extra power supply and have a minimum number of connections;
(b) have a range of at least ±30°, with the highest possible sensitivity;
(c) cause negligible precession of the gyroscope;
(d) have a negligible time constant;
(e) be cheap to produce;
(f) have a high signal/noise ratio.

With reference to (f), it is of interest to note that as the pick-off output would already be a velocity signal, a lower signal/noise ratio could be permitted than on a displacement signal.

The methods available for producing electrically a velocity signal are not numerous; such signals are usually obtained from an electromagnetic generator, and it was decided that such a generator should form the basis of the present pick-off, particularly in view of the above-listed requirements.
Construction

The pick-off consists essentially of

(a) permanent magnet with pole pieces;

(b) armature with pole pieces;

(c) armature coil.

Sketches of the permanent magnet together with its pole pieces are given in Fig.1. The magnet is cylindrical and is made of sintered Alomax 3, polarized in the direction shown. The pole pieces are of sintered soft iron. This unit was made from a single cylindrical piece of sintered material, the majority of the soft iron being removed by turning and the magnet surface cleaned off by grinding. The brass insert facilitates the mounting of the magnet and the light alloy cup provides a protective housing.

The armature consists of a central cylindrical core and two pole pieces as shown in Fig.2. The core forms an integral part of one pole piece, thus leaving only one metal-to-metal joint in the magnetic circuit of the armature. Both core and pole pieces are of Radicmetal.

The coil consists of 2,500 turns of 48 S.W.G. (0.0016 inch) enamelled copper wire. This was former wound, taped and then fitted over the armature core.

Method of Operation

In the following explanation leakage reluctances and magnetic non-linearities are neglected and the reluctances of all ferromagnetic paths are assumed negligible compared with the air gap reluctances.

Fig. 3a shows the armature in its datum position. The gaps A, B, C and D between the armature pole pieces and magnet pole pieces then have approximately equal reluctances and the permanent magnet flux has two similar parallel paths. One is from the N pole piece, via B and A in series to the S pole piece, and the other via D and C. Thus no flux passes through the armature core and hence none links with the coil.

With the armature fully deflected anti-clockwise as shown in Fig. 3b, all the flux from the N pole piece passes across gap D, through the armature core and via gap A to the S pole piece. In this position all the flux links with the coil. Similarly, when the armature is fully deflected clockwise all the flux again links with the coil but in the reverse direction.

With the armature in any intermediate position the flux linking with the coil is proportional to armature deflection, the direction of the flux being determined by the direction of the deflection. Thus when the armature is rotated over its full operating range with a uniform angular velocity, the rate of change of flux linkages with the coil is constant and a constant e.m.f. is generated.

Installation

Before it is fitted to the unit the magnet assembly is magnetised with the aid of the accessories shown in Fig.4. After magnetisation, but before the assembly is withdrawn from the magnetiser, the two armature keepers are placed in position and left until the assembly is installed.
i.e. until the armature provides a low reluctance path for the flux. The light alloy cup is left in position during magnetisation; it is comparatively thin and its reluctance is not significant unless the magnetiser is weak. The non-magnetic spacers prevent compressive loads that occur during magnetisation from distorting the magnet assembly.

To simplify the coil connections the armature is made to serve as a stator and the magnet as a rotor. The inner bore of the armature core is counterbored and threaded to permit the armature assembly to be screwed onto the brass bearing housing, which protrudes slightly from the outer gimbal. Correct orientation of the armature with respect to the outer gimbal is necessary to enable the coil leads to be led away along slots milled in the gimbal, and is obtained by the use of shimming washers between the armature and the gimbal.

The magnet assembly is attached to the inner gimbal bearing spigot by force-fitting the brass insert onto the spigot. In the present application the velocity pick-off is intimately associated with the potentiometer pick-off, the wiper of which is fixed to the magnet assembly. The adhesive plus an anodic film on the light alloy cup provide sufficient insulation between the wiper and the magnet.

Finally the pick-off is subjected to a demagnetising field to ensure stability. The field strength may be somewhat arbitrary but it should definitely be larger than any to which the pick-off is likely to be subjected in practice.

The pick-off is shown installed on the outer gimbal in Fig.5(a). As the pick-off was installed for test purposes only, the leads were brought directly away from the outer gimbal. On a final version slip-rings would of course be necessary.

Although the unit was designed to fit on the outer gimbal to measure inner gimbal velocity, the possibility of fitting it onto the frame to measure outer gimbal velocity was also considered and the unit is shown installed for this function in Fig.5(b). It will be seen that the pick-off has been sandwiched between the standard potentiometer housing and the gyroscope frame. The housing is raised and carried on three supports but is otherwise unmodified. The slip-ring assembly and potentiometer wiper are mounted on top of the magnet assembly, the leads to the slip-rings being brought up through a hole in the centre of the armature and magnet.

The terminal blocks were attached for test purposes only. Also seen in this photograph is a split pulley by which the gimbal was rotated.

6 Test Results
6.1 Resistance and Inductance Measurements
6.11 D.C. resistance 1250 ohms.

6.12 Inductance, measured with a Muirhead Type D-197-A impedance bridge at 1000 c/s.

(a) Armature in datum position as in Fig.3(a):-

\[ L = 0.63 \text{ henry at a } Q \text{ of 1.58.} \]

Therefore the total A.C. resistance at 1000 c/s is 2500 ohms, and the iron loss resistance is \((2500 - 1250) = 1250 \text{ ohms.}\)
(b) Armature as in Fig. 3(b):

\[ L = 0.53 \text{ henry at a } Q \text{ of } 1.5. \]

Therefore the total A.C. resistance at 1000 c/s is 2220 ohms, and the iron loss resistance is \((2220 - 1250) = 970 \text{ ohms.}\)

6.2 Initial Checks on Sensitivity

These were made with the pick-off installed to measure inner gimbal velocity.

Fig. 6(a) shows the equivalent circuit of the pick-off connected to an oscillograph and Fig. 7 shows typical records. Curve 1 of Fig. 8 shows the relevant part of a typical record plotted out on a larger scale. Curve 1 of Fig. 9 shows the relationship between pick-off output and velocity, the slope of the curve showing the sensitivity to be about

\[
\frac{0.02 \text{ volts}}{\text{radian/second}}.
\]

6.3 Tests With Increased Magnetisation

The above-mentioned sensitivity was unexpectedly low and it was thought that this may have been due to inadequate magnetisation. The pick-off had been magnetised after installation, and apparently the reluctances of the air gaps that were necessarily present between the pick-off and magnetiser pole pieces were too large to permit saturation of the magnet. It was at this stage therefore that the accessories shown in Fig. 4 were introduced and saturation thus assured by magnetising prior to installation. Curve 2 of Fig. 9 illustrates subsequent test results, the sensitivity of \(0.038\) \text{ volts radian/second}\) representing an increase of 90%.

6.4 Tests With Increased Magnet Thickness

The sensitivity was further increased by fitting an assembly in which the magnet thickness had been increased from 0.030 inch to 0.050 inch, but which was otherwise unmodified. The new value of sensitivity was found to be about \(0.064\) \text{ volts radian/second}\), a further increase of 60%. This magnet was not retained for further tests however as it had been slightly cracked whilst being machined.

6.5 Unit Installed to Measure Outer Gimbal Velocity

To be certain that the unit could also be used to measure outer gimbal velocity it was refitted as shown in Fig. 5(b), and another series of test runs then made, with the results shown by curve 3 of Fig. 9.

7 Discussion of Results

7.1 Resistance and Inductance Measurements

These were made primarily to enable an approximate equivalent circuit to be drawn for the pick-off.

The magnitude of the inductance is seen to depend upon the position of the armature and although some difference may be expected it is not quite clear why the inductance should be larger with the armature in its datum position. A possible explanation is that with the armature in this position the flux set up by the measuring equipment has a return path.
confined to soft iron, whereas with the armature fully deflected this flux must traverse the permanent magnet material, which has a lower permeability than the iron. The effective A.C. resistance is also somewhat lower when the measurement flux is forced to return via the permanent magnet material, presumably due to the higher resistivity of the material.

It is considered that the iron loss resistance, which was found to be about 1000 ohms at 1000 c/s, may be ignored when considering the equivalent circuit of the pick-off.

7.2 Natural Frequency, Resonant Frequency and Time Constant

Fig. 6(a) gives a reasonably complete equivalent circuit for the pick-off under typical working conditions and Fig. 6(b) gives an approximate general circuit. Referring to the latter we see that we may regard the pick-off as a perfect generator combined with an output circuit having a transfer function given by:

\[
\frac{e_0}{e_1} = \frac{R}{pCR+1} + pL
\]

Simplifying:

\[
\frac{e_0}{e_1} = \frac{1}{LC} \times \frac{1}{\left(p^2 + \frac{p}{RC} + \frac{1}{LC}\right)}
\]

The term within the brackets is recognisable as a standard form which shows that the circuit will have a natural frequency given by

\[
f_d = \frac{1}{2\pi} \sqrt{\frac{1}{LC} - \frac{1}{4\pi C^2}}
\]

and an exponential decay factor \(e^{-\frac{t}{2RC}}\) i.e. a time constant of \(2RC\).

To find the resonant frequency we consider steady-state conditions only. Replacing \(p\) by \(j\omega\) in equation (1), where \(\omega = 2\pi f\), and rationalising:

\[
\frac{e_0}{e_1} = \frac{R}{\sqrt{[(R - \omega^2 LCR)^2 + \omega^2 L^2]}}
\]

It can be shown that the resonant frequency, i.e. the frequency at which \(\frac{e_0}{e_1}\) is a maximum, is:

\[
f_r = \frac{1}{2\pi} \sqrt{\frac{1}{LC} - \frac{1}{2R^2C^2}}
\]
The above expressions for natural and resonant frequencies imply an oscillatory circuit, i.e. that \( \frac{1}{L} > \frac{1}{2RC} \) or \( \frac{1}{2RC} \) respectively.

Considering the latter, the condition is that \( \frac{L}{2RC} < 1 \). Substituting values shown in Fig.6 for \( L, C \) and \( R \) we get \( \frac{L}{2RC} = 0.013 \), thus validating the assumption.

Evaluation of equations (3) and (5) gives 20,450 and 20,400 c/s respectively, while the time constant \( 2RC \) equals \( 9.5 \times 10^{-5} \) seconds. It will be appreciated that all these values are of such a magnitude that they may be neglected entirely.

While this conclusion is of course true only for the particular load condition of Fig.6, it is thought that similar conditions could usually be obtained in practice.

7.3 Sensitivity

As shown in Appendix I the theoretical sensitivity \( S \) of the pick-off is given approximately, in o.g.s. units, by

\[
S = \frac{NFM}{a} \left\{ \frac{G + H(1 + a^2)}{[G + H(1 - a^2)]^2} \right\} \frac{abvols}{\text{radian/second}}
\]

where \( G = (R_p + R_M + R_d) \)

\( H = \frac{R_p R_M}{R_d} \)

\( N = \text{number of turns on the coil} \)

\( R_M = \left( \frac{R_L}{R_M + R_L} \right) P_M \) \text{ gilberts} \)

\( R_M = \frac{R_L R_M}{R_L + R_M} \) \text{ gilberts} \)

\( R_L = \text{total leakage reluctance, gilberts} \)

\( R_p = \text{armature reluctance, gilberts} \)
\[ R_d = \text{datum gap reluctance, gilberts maxwell} \]
\[ = \frac{g}{\alpha \omega r} \text{, where } g, \alpha, \omega, \text{ and } r \text{ are constants dependent upon the geometry of the pick-off. They are defined in Fig. 12.} \]
\[ \omega t = \text{displacement of the armature, in radians. The pick-off has a range of } \pm \alpha \text{, therefore } \omega t_{\text{max}} = \alpha \text{.} \]
\[ a = \frac{\omega t}{\alpha} = \text{fractional displacement of the armature} \]
\[ F' = \text{recoil m.m.f. of the magnet} \]
\[ R'_M = \text{recoil reluctance of the magnet} \]

In the following paragraphs equation (6) is considered firstly as a general expression for the sensitivity of a pick-off of this type and secondly, in relation to the present pick-off.

7.31 General Sensitivity Equation

One characteristic of importance illustrated by equation (6) is the dependence of the sensitivity on 'a', the fractional displacement of the armature. The degree of dependence is in turn governed primarily by \( R_p \), the armature reluctance. When \( R_p = 0 \) the sensitivity is completely independent of armature position.

If then a constant sensitivity is required, the armature should, if possible, be fabricated from a single piece of metal: even faced metal-to-metal joints sometimes contribute an appreciable reluctance.

The degree to which sensitivity is dependent on armature displacement is also governed to some extent by \( R'_M \) and \( R_d \). \( R'_M \) should be small, thus \( R_p \) and \( R_d \) should be small. For a given magnet material \( R'_M \) is, however, invariable. Although from this point of view \( R_d \) should be as large as possible, this, as we shall see later, is undesirable in other respects.

As an alternative it may be an advantage in some control systems to have a velocity term which increases with armature displacement. Figs. 7 and 8 show experimental and theoretical examples of the type of increase that may be expected. It will be seen that not only does the sensitivity increase with displacement but that the rate of change of sensitivity also increases. If then such increases are desirable they may be achieved by controlling the armature reluctance. This is not likely to modify substantially the sensitivity at the datum position, as we may see by putting \( \omega t = 0 \) in equation (6):-
and noting that in a practical case it is probable that $R_p \ll R_M$ and $\frac{R_p}{R_d} \ll 1$.

The ratio of the two extreme values of sensitivity may be obtained by making the appropriate substitutions in equation (6):-

$$\frac{S_{t=\alpha}}{S_{t=0}} = (1 + k) - (1 + 2k) \quad (8)$$

where

$$k = \frac{R_p R_M}{R_d (R_p + R_M + R_d)} = \frac{H}{G}$$

For further discussion it is convenient to assume that a constant sensitivity is required and that $R_p$, the armature reluctance, is negligibly small. Putting $R_p = 0$ in equation (7) gives:

$$S_{R_p=0} = \frac{NF_M}{\alpha} \left( \frac{1}{R_M + R_d} \right) \quad (9)$$

By making use of the substitutions for $F_M$ and $R_M$ given under equation (6), equation (9) may be rewritten:

$$S_{R_p=0} = \frac{NF'_M}{\alpha \left[ R_M' + R_d \left( 1 + \frac{R'_M}{R_L} \right) \right]} \quad (10)$$

If, as may be expected in practice, $R_d \ll R_M'$, then equation (10) illustrates three noteworthy points:

(a) An appreciably low leakage reluctance will not greatly affect the sensitivity.

(b) If, on the other hand, the leakage reluctance is high, large percentage differences in the radial length of the working gap will have little effect on sensitivity.
(c) If the leakage reluctance is high the sensitivity may be expressed without any great error as:

\[ S_{R_f=O, R_L=\infty} = \frac{N\phi_M'}{2\pi R_M} = \frac{N}{\alpha} \phi_M \]  

\[ S = \]  

(11)

where \( \phi_M \) = total magnet flux.

If the pick-off volume is limited, so that the magnet thickness may be increased only at the expense of armature coil thickness, then equation (11) is useful as a guide to the optimum proportions of copper and magnet material. This optimum may be found by determining the maximum value of the product \( N\phi_M \). For a given magnet material \( \phi_M \) is proportional to the cross-sectional area of the magnet, and if the effective width of the magnet is assumed constant then it is proportional to thickness.

Hence,

\[ S_{R_f=O, R_L=\infty} = K_1 T_M T_C \]  

\[ T_M + T_C = K_2 \]  

(12)

where \( T_M \) = magnet thickness

\( T_C \) = coil thickness

\( K_1, K_2 \) are constants

The maximum value of sensitivity as determined from equations (11) and (12) occurs when \( T_M = T_C \).

7.32 Application of the General Equation to the Present Pick-off

Consider now the above discussion in relation to the present pick-off.

The theoretical sensitivity may be obtained by inserting into equation (6) the values appropriate to the present unit, which are:

\[ N = 2500 \text{ turns} \]
\[ g = 0.006 \text{ cm} \]
\[ W = 0.4 \text{ cm} \]
\[ r = 0.9 \text{ cm} \]
\[ \alpha = 0.75 \text{ radian} \]
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\[ B'_{M} = 3720 \text{ gilberts, assuming that the effective length of the magnet } = 1 \text{ cm} \]

\[ B'_{M} = 3.48 \text{ gilberts/maxwell. This is for a magnet of 0.0762 cm (0.030 inch) thickness and assumes an effective magnet cross section of 0.1 sq.cm.} \]

\[ R_{P} = 0.00319 \text{ gilberts/maxwell} \]

\[ R_{L} = \text{ i.e. leakage fluxes are neglected} \]

The values for \( B'_{M} \) and \( B'_{M} \) are of necessity very approximate as they involve estimations of the effective length and cross-section of the magnet. The value for \( B'_{M} \) also involves the estimation of the position of the recoil loop. The value for \( R_{P} \) assumes that the effective length of the air gap at the metal-to-metal joint is 0.0019 cm, a value based on some figures quoted by Roters1.

Curve 2 of Fig. 8 illustrates graphically the evaluation of equation (6) for various armature deflections. The output with the armature in its datum position is calculated to be 1.7 volts, which is in good agreement with the experimental value of 1.8 volts. This must be considered somewhat fortuitous, however, in view of the approximations for \( B'_{M} \), \( B'_{M} \) and \( R_{L} \) noted above.

Asymmetry in the experimental curve (curve 1 of Fig. 8) is probably due to dimensional asymmetry in the air gaps. Nevertheless, it may be seen that over a range of \( \pm 30^\circ \), the output may be assumed to be proportional to velocity within \( \pm 6.5\% \).

The fact that the output from the pick-off is less dependent on armature position than the calculations predict, appears to show that the value assumed for armature reluctance was too high. On the other hand, using this assumed value for armature reluctance, evaluation of equation (8) for the present unit gives a ratio of sensitivities:

\[
\frac{S}{\omega=\alpha} \div \frac{S}{\omega=0} = 1.5
\]

(13)

The average experimental value, however, obtained from a number of curves similar to those of Fig. 7, was found to be 1.8, which tends to show that the assumed armature reluctance was too low. Although there appears at first to be a contradiction here, it should be remembered that as a \( +1 \), fringing and changes in the flux density in the air gap can no longer be neglected.

Consider now the simplified expression of equation (10). \( B'_{M} \) and \( R_{d} \) are taken as 3.48 and 0.022 c.g.s. units respectively. Thus:

(a) The difference in sensitivity between the ideal case when \( R_{L} = \infty \) and the case when \( R_{L} \) is as low as, say, 1.0 c.g.s. unit, is \( < \% \).

(b) If the leakage reluctance is considered high enough to be neglected, then doubling the present radial length of air gap (0.006 \( \rightarrow 0.012 \text{ cm} \)) produces \( < 1\% \) change in sensitivity.

-13-

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Magnets of two different thicknesses, 0.030 inch and 0.050 inch, were tried without modifying the coil thickness. The thicker one was found to give a proportional increase in sensitivity (see Section 6.4).

The effective thickness of the armature coil was about 0.07 inch. If the available space was shared equally between magnet and coil then the expected increase in sensitivity would be $100 \left( \frac{0.050}{0.07} \right) \%$.  

7.4 Reaction Between Moving Parts

As magnetic circuits tend to rearrange themselves so as to include the least reluctance, the armature of the present pick-off will tend to return to its datum position. As shown in Appendix I, the torque involved is given approximately, in c.g.s. units, by

$$\text{Torque} = \frac{a R_p E^2}{4 \pi e K_5} \left( 1 + \frac{R_p}{R_d} \right) \text{dyne cm} \quad (14)$$

where

$$K_5 = \left[ \frac{R_d + R_p + R_M + \frac{R_p R_M}{R_d} \left( 1 - a^2 \right)} {R_d} \right] ,$$

and the other symbols are as defined in Section 7.3, pages 9 and 10.

It may be seen from equation (14) that the torque is zero when the armature is in its datum position and that it increases almost in proportion to the deflection.

The significance of the armature reluctance is further emphasized by equation (14): as the armature reluctance approaches zero so the torque approaches zero and the torque displacement characteristic approaches a straight line. The slope of this characteristic may be considered as the stiffness of an "electrical spring".

A guide to the maximum torque that will be exerted by the present pick-off may be obtained by putting $a = 1$ in equation (14), together with the values given in Section 7.32. This gives a torque of 382 dyne cm and it can be shown that, assuming a rotor speed of 24,000 r.p.m., this will cause the Mark 4 gyroscope to precess at about 10°/minute.

Unfortunately it was not possible to obtain a reliable check on this figure as the performance of the gyroscope without the pick-off was not known sufficiently accurately. It should be noted that

(a) this figure is a maximum and it assumes a value for $R_p$ which may be too high (see Section 7.32);

(b) the precession rate may be reduced by reducing $R_p$.

7.5 Operating Flux Densities

From equation (I.16) of Appendix I we may see that the flux through the armature in its fully deflected position is
Substituting the values assumed in Section 7.32:-

\[ \phi_p = \frac{F_M}{R_p + R_m + R_d} \text{ maxwells} \]  

This flux passes through an armature of varying cross-section, the smallest value being about 0.18 sq. cm, giving a maximum flux density:-

\[ B_p(\text{max.}) = \frac{1060}{0.18} = 5900 \text{ gauss} \]  

This value is relatively low and it is reasonable to suppose that a further increase in sensitivity could be obtained by reducing the cross-section of the ferromagnetic parts and correspondingly increasing the number of turns on the coil. A consideration of the present pick-off leads to the conclusion that such a redesign would increase the sensitivity by 20 - 30%.

A similar theoretical check of various other parts of the magnetic circuit shows that there would be little extra to gain from further dimensional adjustments.

8 Applications

8.1 Velocity Pick-off

The demand for a pick-off of this type is dependent initially on the demand for D.C. servo-systems. When such systems are required it may be worth giving consideration to the following methods of producing a velocity signal:

(a) The use of a gyroscope having a mechanical response proportional to velocity (i.e. a 'rate' gyro) and fitted with a displacement pick-off.

(b) The use of a shaping circuit to operate on a displacement signal obtained from a free gyroscope.

(c) The use of a velocity pick-off fitted to a free gyroscope.

A rigid comparison of these methods is impossible as much depends on the particular circumstances in which they are required to function, but some features may be noted here:

Method (a)

One disadvantage of this method is that normally two gyroscopes and two pick-offs are required, one of each to provide a displacement signal and one of each to provide a velocity signal. Another disadvantage is that the final value of the velocity term is reached only after a significant delay.
Consider for example the response of a rate gyroscope to a step function velocity input. It can be shown that the build up of the velocity term is of the form \( e^{-h \omega_n t} \), where \( h \) is the damping factor expressed as a ratio of critical damping and \( \omega_n \) is the undamped natural frequency of the gimbal suspension. To reduce the delay \( \omega_n \) should be as high as possible, but increases in \( \omega_n \) are accompanied by rapid decreases in the magnitude of the output, which is inversely proportional to \( \omega_n^2 \). In turn, such decreases accentuate the difficulties of providing a suitable pick-off, particularly one which is D.C. energised. Although the delay may be reduced by increasing \( h \), it is not possible to increase it much above 0.7 without appreciably reducing the operating frequency range of the gyroscope. In addition, when \( h < 1 \) the velocity signal will have a spurious oscillation superimposed on it.

One advantage of this method is that the displacement signal remains unadulterated.

Method (b)

This method has the advantage of needing only one gyroscope, one pick-off and a simple shaping circuit. If we regard the output of the shaping circuit as consisting of two components, one proportional to displacement and one proportional to velocity, then it is found that the displacement term is attenuated and the velocity term subject to a delay.

Consider for example the elementary circuit of Fig.13, and assume again a step function velocity input. The displacement term is attenuated by a factor \( G \), where \( G = \frac{R_1}{R_1 + R_2} \), and the build up of the velocity term is of the form \( e^{-\frac{t}{T_1}} \), where \( T_1 = \frac{1}{G R_2} \). If an attempt is made to reduce the delay by reducing \( T_1 \) it is found that the magnitude of the velocity term is reduced, while reductions in \( G \) reduce both displacement and velocity terms.

As noted earlier another disadvantage of this method is that the signal/noise ratio is reduced by the shaping circuit.

Method (c)

Assuming again that both velocity and displacement signals are required this method calls for one gyroscope and two pick-offs.

It may be difficult to obtain a range of \( > 90^\circ \) for the velocity pick-off and this may be a disadvantage for some applications. Also, the complexity of the gyroscope is somewhat increased by the addition of the second pick-off, and in the present instance the magnitude of the velocity term may be considered too small. It is reasonable to suppose, however, that in many applications space would permit the volume, and hence the sensitivity, to be increased to several times the present figure. Alternatively, as the pick-off output is reasonably free from noise it may be possible to amplify it.

Consideration of current rate gyroscopes and shaping circuits shows that the time delays and other undesirable characteristics of methods (a) and (b) are of significance. With the present velocity pick-off, however, it was shown in Section 7.2 that, although strictly at least a second order system, provided that it is loaded with a high impedance it may be regarded as a pure generator.
As with method (a) the displacement signal remains unadulterated.

8.2 Torsional Vibration Pick-up

Another possible use for the magnetic circuit arrangement used in this unit is as the basis for a torsional vibration pick-up. The output of 0.064 \( \text{volts} \text{ radian/second} \) of the present unit compares very favourably with the output of 0.090 \( \text{volts} \text{ radian/second} \) of the current R.A.E. torsional vibration pick-up, although the volume of the latter is many times as great. The range of the present unit is also several times greater but it is doubtful whether this increase is in itself of much value, although it may be possible to sacrifice it to the advantage of other characteristics.

8.3 Torque Motor

The transducing action of the pick-off is of course reversible and by passing a current through the coil the unit may be used as a wide-angle miniature torque motor.

Equation (1.32) of Appendix I provides a general expression for the torque output of a device of this nature. An expression for the torque with the armature central may be obtained by putting \( a = 0 \) in this equation:

\[
T_{a=0} = \frac{1}{4 \pi a} \left( \frac{P M P P}{P P + R M + R d + \frac{R M P}{R d}} \right) \text{dyne cm} \tag{18}
\]

This may be evaluated for the present unit by assuming various currents and substituting the values given in Section 7.32 for the other parameters. As a check, assuming a current through the armature coil of, say, 9mA, we get:

\[
T_{a=0} = 2.85 \text{ grms.wt.cm}
\]

In practice a current of 9mA was found to produce a torque of between 3 and 3.5 grms.wt.cm.

It was noted in Section 7.32 that the value assumed for armature reluctance appeared to be too high and it is of interest to note that a lower assumed value would also give a more accurate prediction of \( T_{a=0} \). It should also be noted that equation (18) assumes linear flux-current relationships.

As a torque of 3.0 grm.cm appears small it is pertinent here to point out that

(a) the volume of the unit is less than 3.0 cm.
(b) as a torque motor the operating range is up to \( \pm 45^\circ \);
(c) the torque is obtained with a power dissipation of 0.1 watt;
(d) if the unit is fed from a source of zero internal impedance the time constant involved, as given by \( \frac{L}{R} \) is about 0.5 millisecond.
The coil may be centre-tapped to permit the unit to be used as a differential torque motor.

Fig. 14 shows a suggested arrangement for a torque motor. It consists essentially of two units "back to back", one advantage of this arrangement being that with the magnet positioned in the centre of the device, the leakage flux is reduced to a minimum.

9 Conclusions

Development of this unit is as yet incomplete but it appears to present an efficient magnetic circuit arrangement for use in wide-angle transducers.

Considering the unit as a velocity pick-off, it has been noted that it should be possible, by simple dimensional changes, to increase the sensitivity and range, and reduce the variation of sensitivity with angle. Alternatively the sensitivity may be made to increase as the angular displacement increases by specifically introducing reluctance into the armature.

Compared with a wire-wound potentiometer it has a very smooth output. This output may therefore be operated on with less deterioration in the signal/noise ratio.

When the armature is displaced from its datum position a small restoring torque is exerted on it. This is not likely to be serious, but should it appear so the torque may be reduced by reducing the armature reluctance.

It has not yet been possible to test the effect of specific dimensional asymmetries in the air gaps, or of missile accelerations.

As a torsional vibration pick-up or as a torque motor this unit appears to have advantages over existing versions.

The bi-metal sintering process reduces leakage and unwanted circuit reluctances, but the method used to manufacture the present magnet assembly is not completely satisfactory for such small units, as it calls for unusual machine shop techniques.

10 Acknowledgement

Thanks are due to Mr. Chalmers of G.W. Drawing Office for assistance in the design of this prototype.
**Technical Note No. G.W. 231**

**REFERENCE**

<table>
<thead>
<tr>
<th>No.</th>
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**Attached:**

- Appendix I
- Drg. Nos. GW/P/4172 to 4176 and 4178 to 4183
- Neg. Nos. 103927 to 103928
- Detachable Abstract Cards

**Advance Distribution:**

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-19-
APPENDIX I

Sensitivity and Torque Calculations

In the following paragraphs recoil parameters are defined and used to establish equations for armature and air gap fluxes. From these equations expressions are derived for sensitivity and torque. C.g.s. units are used unless otherwise stated.

To ensure stability it is usual to subject a permanent magnet to a demagnetising field considerably greater than any it is likely to encounter in practice. The magnet is then in effect working on a recoil loop.

The magnets used in this pick-off were of sintered Alcanax 3 and the relevant quadrant of the B-H curve for this material is given in Fig.10(a). This curve may be used for a magnet of specific dimensions by converting it to a flux-m.m.f. curve. Fig.10(a) also shows a typical recoil loop. The slope of the mean value of this loop, i.e. the slope of C.D., is roughly the same as that of the demagnetisation curve at $H=0$. The loop may without any great loss of accuracy be replaced by the line C.D. and the projection of this line onto the m.m.f. scale provides a useful fictitious parameter, the "recoil open circuit" m.m.f. for the magnet. This is illustrated in principle by Fig.10(b). Thus as long as the magnet is operating at some point along the line C.D., it may be considered as a linear element with a constant internal permeance given by the slope of C.D., and a "recoil" m.m.f. given by C.E.

We thus have a direct analogy with electrical circuits and using electrical symbols the magnetic circuit of the pick-off may be represented as shown in Fig.11(a).

It should be noted that Fig.11(a) is an idealized circuit in which air gap flux densities are assumed to be independent of armature position and uniform throughout each gap. Also the reluctances of the ferrous parts of the circuit are considered negligible compared with those of the air gaps, and flux is assumed to be proportional to current.

Although nominally the armature itself contains no air gap, it cannot be ignored as it contains a metal-to-metal joint which may have an appreciable reluctance.

In Fig.11(a):

$R_M' = \text{recoil m.m.f. of the magnet}$

$R_M = \text{recoil reluctance of the magnet}$

$R_L = \text{total leakage reluctance}$

$R_P = \text{reluctance of the armature}$

$R_{A,B,C,D} = \text{reluctances of the working gaps A, B, C and D (see Fig.3)}$
Assuming symmetry, \( R_C = R_D \) and \( R_C = R_A \). Also, \( R_A \) may be taken into account by the use of a modified recoil m.m.f. (\( F_M \)) and reluctance (\( R_M \)). This permits the circuit to be simplified to that of Fig. 11 (b) in which

\[
F_M = \left( \frac{R_L}{R_M + R_L} \right) F_M \tag{I.1}
\]

and

\[
R_M = \frac{R_L R_M}{R_M + R_L} \tag{I.2}
\]

Also,

\[\phi_A = \text{flux through } R_A\]

\[\phi_B = \text{flux through } R_B\]

This circuit also includes an additional element, \( F_P \), which represents the m.m.f. created when a current flows through the armature coil, and is introduced in order to generalise. When the unit is considered as a velocity pick-off \( F_P \), provided the load on the pick-off is of high impedance.

Assuming the existence of circulating fluxes \( \phi_1, \phi_2, \) and \( \phi_3 \), we may write down the following for Fig. 11 (b):

\[
\phi_A = \phi_3 - \phi_1 \tag{I.3}
\]

\[
\phi_B = \phi_2 - \phi_1 \tag{I.4}
\]

\[
\phi_F = \phi_3 - \phi_2 \tag{I.5}
\]

\[
\phi_1 R_M + (\phi_1 - \phi_3) R_A + (\phi_1 - \phi_2) R_B = F_M \tag{I.6}
\]

\[
\phi_2 R_A + (\phi_2 - \phi_1) R_B + (\phi_2 - \phi_3) R_2 = F_P \tag{I.7}
\]
These equations may be solved to give

$$\varphi_A = -1 \left\{ \frac{F_F (R_M + R_B) + F_M (R_P + R_B)}{K_1} \right\}$$  \hspace{1cm} (I.9)

$$\varphi_B = \frac{F_F (R_M + R_A) - F_M (R_P + R_A)}{K_1}$$  \hspace{1cm} (I.10)

$$\varphi_P = \frac{F_M (R_A - R_B) - F_F (R_A + R_B + 2 R_M)}{K_1}$$  \hspace{1cm} (I.11)

where

$$K_1 = 2 (R_A R_B + F_P R_M) + (F_F + R_M) (R_A + R_B)$$

We will first consider the unit as a velocity pick-off and proceed to find a general expression for its sensitivity.

When \( F_F = 0 \),

$$\varphi_P = \frac{F_M}{K_1} (R_A - R_B) \text{ maxwells}$$  \hspace{1cm} (I.12)

Expressions for the gap reluctances, \( R_A \) and \( R_B \), in terms of the pick-off dimensions may be found by referring to Fig. 12. From this:

$$R_d = \frac{\varepsilon}{W r a} \text{ gilberts maxwell}$$  \hspace{1cm} (I.13)

$$R_A = \frac{\varepsilon}{W r (\alpha + \theta)} \text{ gilberts maxwell}$$  \hspace{1cm} (I.14)

$$R_B = \frac{\varepsilon}{W r (\alpha - \theta)} \text{ gilberts maxwell}$$  \hspace{1cm} (I.15)
Making use of these substitutions, equation (1.12) becomes

\[ \Phi_P = \frac{\theta F_M}{a \left[ (R_P + R_M + R_d) + \frac{R_P R_M}{R_d} \left( 1 - \frac{\theta^2}{a^2} \right) \right]} \text{ maxwells (I.16)} \]

If \( N \) = number of turns on armature coil and \( \theta = \omega t \), then the total flux linkages are

\[ N \omega t \frac{P}{M} \]

The e.m.f. generated is given by \( \frac{d}{dt} (N\Phi_p) \), and the sensitivity \( S \), by

\[ S = \frac{1}{\omega} \frac{d}{dt} (N\Phi_p) \text{ abvolts radian/second (I.18)} \]

Evaluation of equation (I.18) gives

\[ S = \frac{NEM}{a} \left\{ \frac{G + H (1 + a^2)}{[G + H (1 - a^2)]^2} \right\} \text{ abvolts radian/second (I.19)} \]

where \( G = (R_P + R_M + R_d) \)

\[ H = \frac{R_P R_M}{R_d} \]

\[ a = \frac{\omega t}{a} = \text{ fractional displacement of the armature} \]

We next proceed to find a general expression for the torsional reaction between the moving parts.

It can be shown \(^1\) that the tangential force at a variable air gap such as exists in the present unit is

\[ P = \frac{F^2 M}{8 \times g} \text{ dynes (I.20)} \]

where \( F = \text{ m.m.f. across the gap} \).
Thus referring to Fig.3, the force at gaps A and B will be

\[ F_A = \frac{P_A^2 W}{8\pi g} \text{ dynes} \]  \hspace{1cm} (I.21)

\[ F_B = \frac{P_B^2 W}{8\pi g} \text{ dynes} \]  \hspace{1cm} (I.22)

The nett force,

\[ P_B - P_A = \frac{W}{8\pi g} (P_B^2 - P_A^2) \text{ dyne cm} \]  \hspace{1cm} (I.23)

and the nett torque,

\[ r(P_B - P_A) \text{ dyne cm} \]  \hspace{1cm} (I.24)

Owing to the existence of a similar torque at gaps C and D the total torque \( T \) will be

\[ T = 2r(P_B - P_A) = \frac{rW}{4\pi g} (P_B^2 - P_A^2) \text{ dyne cm} \]  \hspace{1cm} (I.25)

Also,

\[ F_B = \varphi_B R_B \text{ gilberts} \]  \hspace{1cm} (I.26)

and

\[ F_A = \varphi_A R_A \text{ gilberts} \]  \hspace{1cm} (I.27)

It is convenient at this stage to express \( R_A \) and \( R_B \) in terms of \( R_d \), the reluctance of each gap with the armature at its datum position, and \( \frac{\theta}{a} \), the fractional displacement of the armature. Thus using equation (I.13), equation (I.14) becomes

\[ R_A = \frac{g}{\omega \left(1 + \frac{\theta}{a}\right)} = \frac{R_d}{(1 + a) \text{ maxwell}} \]  \hspace{1cm} (I.28)

where \( a = \frac{\theta}{a} \).
Similarly equation (1.15) becomes

\[ R_B = \frac{R_d}{(1 - a)} \]  \hspace{2cm} (I.29)

Substituting equations (I.10), (I.9), (I.28) and (I.29) into (I.26) and (I.27):

\[ F_B = \frac{1}{K_2} \left[ \frac{F_P R_d}{(1-a)} \left[ R_M + \frac{R_d}{(1+a)} \right] - \frac{F_M R_d}{(1-a)} \left[ R_P + \frac{R_d}{(1+a)} \right] \right] \]  \hspace{2cm} (I.30)

\[ F_A = \frac{-1}{K_2} \left[ \frac{F_P R_d}{(1+a)} \left[ R_M + \frac{R_d}{(1-a)} \right] + \frac{F_M R_d}{(1+a)} \left[ R_P + \frac{R_d}{(1+a)} \right] \right] \]  \hspace{2cm} (I.31)

where

\[ K_2 = 2 \left[ \frac{R_d^2}{(1+a)(1-a)} + \frac{R_P R_M}{R_d} \right] + \left( R_P + R_M \right) \left[ \frac{R_d}{(1+a)} + \frac{R_d}{(1-a)} \right] \]

Substituting equations (I.13), (I.30) and (I.31) into equation (I.25) and simplifying:

\[ T = \frac{K_3 K_4}{4\pi a H} \left( \frac{F_M F_P \left[ 1 + a^2 K_3 K_4 \right] - a \left[ F_M^2 K_3 + F_P^2 K_4 \right]}{\left[ 1 - a^2 K_3 K_4 \right]^2} \right) \]  \hspace{2cm} (I.32)

where

\[ H = \frac{R_P R_M}{R_d} \]

\[ K_3 = \frac{R_P}{R_d + R_M} \]

\[ K_4 = \frac{R_M}{R_d + R_P} \]

This is the complete expression for torque, applicable when the unit is used as a torque motor. If \( R_p \) may be neglected it may be simplified to

\[ T_{R_p=0} = \frac{1}{4\pi a (R_d + R_M)} \left[ R_M F_P - a F_P^2 \frac{R_M}{R_d} \right] \]  \hspace{2cm} (I.33)

-25-
Equations (I.32) and (I.33) may also be used to calculate torque reaction if it is required to use the unit as a rate pick-off loaded with a low impedance. For a velocity \( \omega \):

\[
F_p = \frac{S \omega \frac{\pi}{2}}{Z} \times \frac{4\pi}{9} \text{ gilberts} \tag{I.34}
\]

where \( Z = \) total resistance in armature coil circuit in ohms.

Putting \( F_p = 0 \) gives the following expression for torque suitable for use when the unit is used as a rate pick-off loaded with a high impedance:

\[
T_{F_p=0} = \frac{a R_p R_m^2}{4 \pi a K_5^2} \left( 1 + \frac{R_p}{R_d} \right) \text{ dyne cm} \tag{I.35}
\]

where

\[
K_5 = \left[ R_d + R_p + R_m + \frac{R_m R_p}{R_d} (1 - a^2) \right]
\]

Note that \( T_{F_p=0} \to 0 \) as either \( R_p \) or \( a \to 0 \). Equation (I.35) may also be rearranged to provide an expression which may be regarded as the stiffness of the "electrical spring":

\[
\frac{T_{F_p=0}}{a^2} = \frac{R_p R_m^2}{4 \pi a^2 K_5^2} \left( 1 + \frac{R_p}{R_d} \right) \text{ dyne cm \ radian} \tag{I.36}
\]

Finally, attention is drawn to the assumptions inherent in the formulae established above. These formulae must therefore be used with discretion if quantitative accuracy is required.
FIG. I(a-d). MAGNET ASSEMBLY.
FIG. 2 (a-c).

(a) General appearance of armature and coil.

(b) Plan view with coil fitted.

(c) Section AA.

N.B. The central core forms an integral part of one pole piece.

FIG. 2 (a-c). Armature assembly.
FIG. 3 (a & b).

(a). ARMATURE IN DATUM POSITION.

(b). ARMATURE AT LIMIT OF WORKING RANGE.

FIG. 3 (a & b). FLUX PATHS THROUGH ARMATURE.
(a). EXPLODED VIEW OF ACCESSORIES.

(b). ELEVATION OF MAGNET ASSEMBLY POSITIONED FOR MAGNETISING.

FIG. 4(a & b). METHOD OF POLARISING THE MAGNET.
TECH. NOTE: G.W. 231

FIG. 5

a. PICK-OFF INSTALLED TO MEASURE INNER GIMBAL VELOCITY

b. PICK-OFF INSTALLED TO MEASURE OUTER GIMBAL VELOCITY

FIG. 5. PICK-OFF IN POSITION
(a). CIRCUIT FOR PICK-OFF UNDER TEST.

(b). APPROXIMATE GENERAL CIRCUIT.

FIG.6(a & b). EQUIVALENT CIRCUITS.
a. RECORD SHOWING APPROXIMATELY FOUR COMPLETE REVOLUTIONS OF PICK-OFF

N.B. 1. FREQUENCY OF UPPER TRACE ON BOTH RECORDS IS 50 c/s

2. VERTICAL STROKES ON RECORD (b) ARE SPURIOUS

b. RECORD SHOWING APPROXIMATELY 110° OF ROTATION

FIG.7. TYPICAL RECORDS
FIG. 8. TYPICAL PICK-OFF OUTPUT.

GIMBAL VELOCITY = 55 RADS/SEC.

1 = EXPERIMENTAL RESULTS.

2 = CALCULATED RESULT - ARMATURE ASSUMED TO HAVE A VIRTUAL AIR GAP = 0.00075 IN.
FIG. 9.

1. RESULT OF INITIAL TESTS.
2. RESULTS AFTER INCREASED MAGNETISATION.
3. NOMINALLY AS 2 BUT WITH PICK-OFF MEASURING OUTER GIMBAL VELOCITY.

Sensitivity = \tan \alpha

\begin{align*}
\tan \alpha_1 &= 0.02 \quad \text{VOLTS} \\
\tan \alpha_2 &= 0.038 \quad \text{VOLTS} \\
\tan \alpha_3 &= 0.032 \quad \text{VOLTS}
\end{align*}

ANGULAR VELOCITY IN RADS/SEC.

FIG. 9. RELATIONSHIP BETWEEN VELOCITY & PICK-OFF OUTPUT.
MAGNET THICKNESS 0.030 IN.
This may be converted into a flux- M.M.F. curve by multiplying the vertical and horizontal scales by area and length respectively.

(a). Approximate demagnetisation curve for sintered Alcomax 3.

(b) Method of obtaining recoil parameters.

FIG.IO (a & b). Demagnetisation curve.
POLE PIECE RELUCTANCES ARE NEGLECTED

(a). EQUIVALENT CIRCUIT FOR THE PICK-OFF.

THE ADDITIONAL M.M.F. $F_F$ IS INCLUDED IN ORDER TO GENERALISE.
WHEN THE UNIT IS USED AS A RATE PICK-OFF USUALLY $F_F \neq 0$.

(b). MODIFIED CIRCUIT.

FIG.11(a & b). EQUIVALENT MAGNETIC CIRCUITS.
With the armature in its datum position the reluctance of each gap is given by:

$$ R_d = \frac{g}{W + \alpha} $$

If the armature moves through an angle $\theta$ to the position shown by dotted lines:

$$ R_A = \frac{g}{W + (\alpha + \theta)} $$

$$ R_B = \frac{g}{W + (\alpha - \theta)} $$

**FIG.12. SKETCH DEFINING GAP RELUCTANCES.**
FIG. 12. SKETCH DEFINING GAP RELUCTANCES.

WITH THE ARMATURE IN ITS DATUM POSITION THE RELUCTANCE OF EACH GAP IS GIVEN BY:

\[ R_d = \frac{g}{W + \alpha} \]

IF THE ARMATURE MOVES THROUGH AN ANGLE \( \theta \) TO THE POSITION SHOWN BY DOTTED LINES:

\[ R_A = \frac{g}{W + (\alpha + \theta)} \]
\[ R_B = \frac{g}{W + (\alpha - \theta)} \]
FIG. 13. ELEMENTARY PHASE ADVANCE CIRCUIT.

FIG. 14. SUGGESTED ARRANGEMENT FOR TORQUE MOTOR.
A D.C. ANGULAR VELOCITY PICK-OFF FOR USE ON GYROSOPES

This note describes an angular velocity pick-off for indicating the rate of change of gimballed displacement of a free gyroscope over a range of ±90°, to within ±5%, with a sensitivity of 0.006 volts per radian per second. The pick-off takes the form of a cylinder 0.9 inches in diameter and 0.4 inches long.

The sensitivity could be increased by about 20% without increasing the volume and it could be increased considerably more by a small increase in volume; e.g. a 100% increase in sensitivity could be obtained with about 30% increase in size. Simple modifications may apparently be incorporated.

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to increase the accuracy and range, or to alter the form of indication so that the sensitivity increases with increasing gimbali displacement.

Development is as yet incomplete, but the construction and method of operation are described and preliminary test results are given and discussed.

As the arrangement of the magnetic circuit appears to be an efficient one for wide-angle transducers, brief consideration is given to the possibility of using the unit as a torsional vibration pick-up or as a torque motor.
This document is now available at the National Archives, Kew, Surrey, United Kingdom.

DTIC has checked the National Archives Catalogue website (http://www.nationalarchives.gov.uk) and found the document is available and releasable to the public.

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