AIRCRAFT ELECTRICAL SYSTEM STUDY

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ARMOUR RESEARCH FOUNDATION
ILLINOIS INSTITUTE OF TECHNOLOGY

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Illinois Institute of Technology

April 1952

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Air Research and Development Command
United States Air Force
Wright-Patterson Air Force Base, Ohio
FOREWORD

This report was prepared by Armour Research Foundation of Illinois Institute of Technology to incorporate the results of a study project entitled, "Study, Aircraft Electrical Systems." The study was conducted for the Equipment Laboratory, Directorate of Laboratories, Wright Air Development Center, under RDO 656-2112, "Aircraft Electrical System Evaluation," on Air Force Contract 33-(038)-30511. Mr. C. T. Hackler acted as WADC Project Engineer.

Other personnel of Armour Research Foundation who assisted in the preparation of this report were Messrs. R. M. Bergslien, J. A. Granath, A. K. Hawkes, and R. F. Zenner.

WADC TR 52-49
AIRCRAFT ELECTRICAL SYSTEM STUDY

ABSTRACT

The results of a study to evaluate the performance of aircraft electrical systems are presented in this report. Emphasis has been on the 400-cycle, a-c system. Analytical and experimental studies using the facilities of the Air Force Equipment Laboratory at Wright Air Development Center were made by personnel of the Armour Research Foundation. These studies were made to determine the performance characteristics of the 40 KVA alternators and their associated regulating equipment as well as protective equipment with a view to determine methods of improving the performance and reliability of the electrical system.

From these studies it is concluded that practical improvements in the reliability of the electrical system can be effected, and that detailed study should be directed toward simplifying the system layout and augmenting and improving the means of protecting the electrical system.

PUBLICATION REVIEW

This report has been reviewed and is approved.

FOR THE COMMANDING GENERAL:

M. A. Boushey
Colonel, USAF
Chief, Equipment Laboratory
Directorate of Laboratories

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I. INTRODUCTION

This report summarizes the results of a study of aircraft electrical systems. The objectives of the study were to analyze present standard aircraft electrical systems from the standpoint of overall system reliability.

The work was carried out for the Electrical Branch, Equipment Laboratory, Wright Air Development Center under Contract AF 33(038)-30511. The contract covered a 6 months study ending March 20, 1952.

The emphasis in the study has been on the constant frequency 400 cycle, 200Y/115 volt system. This emphasis resulted from considerations of relative importance of review of the systems and stems from the following considerations:

(a) There has been extensive field experience on 24 volt d.c. systems and has been the subject of several previous studies.
(b) Higher voltage d.c. systems are to be studied under a separate system study contract.
(c) The 400 cycle AC constant frequency system is now coming into considerable use and its application is being extended.

Experimental phases of the program were carried out in the Wright Field Laboratories. Performance of the machines and their controls including constant speed drives was determined under both normal and abnormal conditions. The power generating equipment included two 40 KVA alternators while the utilization devices were represented by resistive and inductive passive loads. Measurements were made with indicating instruments and an oscillograph to determine appropriate parameters for analytical studies. In addition, tests were staged to determine the performance of the system under
conditions which could not be readily analyzed on the basis of system parameters.
II. **PRESENT 400-CYCLE A-C SYSTEMS**

Figures 1 and 2 show complete source sections of an a-c power system including an alternator, controls, protective devices, and leads to the main circuit breaker at the fuselage bus as are used or contemplated at the present time. Two diagrams are shown for each arrangement, an elementary diagram from which sequence of operation can be obtained, and an interconnection diagram which shows the physical configuration of the system.

A. **Alternator**

The alternator, usually with an integral exciter, is designed to operate from a constant speed drive unit and so generates voltage at constant frequency which permits parallel operation with other alternators.

Alternator characteristics for a 40 and a 60 KVA unit are listed in Table I.

---

**Table I**

**ALTERNATOR SPECIFICATIONS**

(From USAF Spec. No. 32509 Amended 13 December 1949 and USAF Exhibit MCREEX-21-48 Revised 16 January 1948)

<table>
<thead>
<tr>
<th>Specifications</th>
<th>Type A-1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Voltage</td>
<td>volts 120/208</td>
</tr>
<tr>
<td>Frequency</td>
<td>cps 400</td>
</tr>
<tr>
<td></td>
<td>Continuous 40</td>
</tr>
<tr>
<td></td>
<td>Five minutes 60</td>
</tr>
<tr>
<td></td>
<td>Five seconds 72</td>
</tr>
<tr>
<td>Power factor</td>
<td>0.75</td>
</tr>
<tr>
<td>Speed, rpm</td>
<td>Nominal 6000</td>
</tr>
<tr>
<td></td>
<td>Range 5100-6600</td>
</tr>
<tr>
<td></td>
<td>Overspeed 9000</td>
</tr>
<tr>
<td></td>
<td>Exciter field voltage volts</td>
</tr>
<tr>
<td>Weight</td>
<td>lbs 80</td>
</tr>
<tr>
<td>Cooling</td>
<td>210 cfm at 6&quot; water</td>
</tr>
</tbody>
</table>

*WADC TR 52-49*
GENERATOR CONTROLS AND COMPLETE DIFFERENTIAL PROTECTION
FIG 2 - ALTERNATOR CONTROLS AND FUSE PROTECTION
Alternator controls are comprised of a voltage regulator and an exciter control relay.

B. Voltage Regulator

The voltage regulator adjusts the carbon stack which is in series with the exciter field to maintain the average line-to-neutral voltage of the three phases at the desired value. The regulator also incorporates a current transformer and mutual reactor which, when interconnected with the regulators of the other alternators on the system, corrects the regulator action to provide equal division of reactive load between alternators. The voltage regulator characteristics are listed in Table II.

Table II

A-C VOLTAGE REGULATOR SPECIFICATIONS
(From MIL-R-5292A 25 April 1951)

<table>
<thead>
<tr>
<th>Specification</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Normal adjusted voltage</td>
<td>208 volts</td>
</tr>
<tr>
<td>Range of adjustment</td>
<td>195 to 215 volts</td>
</tr>
<tr>
<td>Accuracy</td>
<td>2.5%</td>
</tr>
<tr>
<td>Working range of variable resistance</td>
<td>1 to 30 ohms</td>
</tr>
<tr>
<td>Current at min. resistance</td>
<td>8 amps</td>
</tr>
<tr>
<td>Power dissipation, continuous</td>
<td>90 watts</td>
</tr>
<tr>
<td>Weight, max.</td>
<td>12 lbs</td>
</tr>
<tr>
<td>Compensation - 0.5 per unit, zero p.f. lagging, single machine shall cause a drop of 5 to 7% of adjusted voltage</td>
<td></td>
</tr>
</tbody>
</table>

C. Exciter Control Relay

The exciter control relay is provided to open the field circuit either manually or by action of the protective devices. The close coil of the main circuit breaker is in series with a pair of auxiliary ECR contacts so that the breaker can be closed only if the ECR is closed. An engine shutdown switch is also provided to prevent closure of the ECR when the exciter
and alternator are not rotating. A lockout relay is used to prevent recycling in event the circuit breaker is tripped. The ECR is closed by a manually operated switch. The characteristics of the exciter control relay are listed in Table III.

Table III

EXCITER CONTROL RELAY SPECIFICATIONS

(From USAF Spec. No. 32579 Amended 1 May 1947)

<table>
<thead>
<tr>
<th>Specification</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Type</td>
<td>H-1, 2 phase, 400 cycle</td>
</tr>
<tr>
<td>Close coil</td>
<td>200/115 volt</td>
</tr>
<tr>
<td>Operating voltage</td>
<td>12-30 volts d-c</td>
</tr>
<tr>
<td>Time duration to withstand voltage</td>
<td>1 min.</td>
</tr>
<tr>
<td>Trip coil</td>
<td>6-30 volts d-c</td>
</tr>
<tr>
<td>Operating voltage</td>
<td></td>
</tr>
<tr>
<td>Speed of auxiliary contacts to trip circuit breaker</td>
<td>0.008 sec.</td>
</tr>
</tbody>
</table>

D. Circuit Breaker

A circuit breaker connects the alternator to the main bus. The breaker can be tripped by a manually operated switch, by the protective devices, or by the action of the exciter control relay. It is closed by a manually operated switch. The characteristics of the circuit breaker are listed in Table IV.
Table IV
CIRCUIT BREAKER SPECIFICATIONS
(From USAF Spec. No. 32502A 19 Aug. 1946)

| Type A-I Electrically operated, 3 pole, 208/120 volt 400 cycle air circuit breaker |
| Continuous Rating | 120 amp. |
| Five minutes Rating | 165 amp. |
| Five seconds Rating | 245 amp. |
| Interrupting rating | 4000 amps. |
| Interrupting time | 0.01 sec. max. |
| Closing time | 0.05 sec. max. |
| Nominal coil voltage | 24 volts d-c |
| Auxiliary contact rating | a-c 1 amp. at 240 volts |
| | d-c 10 amp. at 30 volts |

Protective devices include an exciter protection relay, differential protection and current limiting fuses. In Fig. 2 the alternator leads have differential current protection from the alternator terminals to the nacelle bus, and overcurrent protection in the form of a fuse mesh network from the nacelle to fuselage bus. The nacelle bus is partially protected by the fuse system. Another system, shown in Fig. 1, is covered by differential protection from the alternator terminals to the main bus. Both systems include exciter protection.

E. Exciter Protection Relay

The exciter protection relay is a thermally actuated device which trips the exciter control relay and circuit breaker when the excitation exceeds a certain limit for a sustained period of time. The characteristics of the exciter protection relay are listed in Table V.
Table V
EXCITER PROTECTION RELAY SPECIFICATIONS
(From USAF Spec. 32664 Amended 25 February 1950)

<table>
<thead>
<tr>
<th>Type A-I</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Contact rating</td>
<td></td>
</tr>
<tr>
<td>Current</td>
<td>10 amps d-c</td>
</tr>
<tr>
<td>Voltage</td>
<td>30 volts d-c</td>
</tr>
<tr>
<td>Nominal rating</td>
<td>44 volts</td>
</tr>
<tr>
<td>Time delay at nominal rating</td>
<td>6 ± 1 sec.</td>
</tr>
<tr>
<td>Minimum operating voltage</td>
<td>20 volts</td>
</tr>
<tr>
<td>Adjustment range</td>
<td>34 to 44 volts</td>
</tr>
</tbody>
</table>

F. Differential Protection Relay

The differential protection relay trips the circuit breaker and exciter control relay on ground or phase-to-phase faults. Table VI lists the characteristics of the differential protection relay.

Table VI
DIFFERENTIAL PROTECTION RELAY SPECIFICATIONS
(From MIL-R-5296 10 April 1950)

<table>
<thead>
<tr>
<th>Type S-1</th>
<th>200/115 volt</th>
<th>400 cycle</th>
</tr>
</thead>
<tbody>
<tr>
<td>Operating time (40 amps differential current)</td>
<td>0.001 sec</td>
<td></td>
</tr>
<tr>
<td>Differential current range</td>
<td>30-4000 amps</td>
<td></td>
</tr>
<tr>
<td>Current capacity of contacts</td>
<td>10 amps</td>
<td></td>
</tr>
</tbody>
</table>

G. Load Transfer Contactor

In systems where the load feeders are multiple channel, the more essential loads are connected to the system by alternate channel feeders switched by a load transfer contactor. In case the feeder supplying the load thru a load transfer contactor fails, the load transfer contactor
automatically switches to the alternate feeder. Table VII lists the characteristics of this device.

Table VII

LOAD TRANSFER CONTACTOR SPECIFICATIONS

(From USAF Exhibit MCREXE-21-59 18 November 1946)

<table>
<thead>
<tr>
<th>Rating</th>
<th>150 amps</th>
</tr>
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<tbody>
<tr>
<td>Current</td>
<td>200 volts</td>
</tr>
<tr>
<td>Voltage</td>
<td>380-460 cps</td>
</tr>
<tr>
<td>Frequency</td>
<td>100 volts (line-to-neutral)</td>
</tr>
<tr>
<td></td>
<td>80 volts (line-to-neutral)</td>
</tr>
<tr>
<td></td>
<td>10%</td>
</tr>
<tr>
<td>Trip rating</td>
<td>0.15 to 0.20 sec.</td>
</tr>
<tr>
<td>pull in</td>
<td>1 ma. maximum</td>
</tr>
<tr>
<td>drop out</td>
<td>0.5 amp at 115 volts</td>
</tr>
<tr>
<td>Overvoltage, continuous</td>
<td></td>
</tr>
<tr>
<td>Time delay</td>
<td></td>
</tr>
<tr>
<td>Holding current, relay</td>
<td></td>
</tr>
<tr>
<td>Holding current, solenoid</td>
<td></td>
</tr>
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III. **SYSTEM PERFORMANCE**

A. **Introduction**

Electric power plays a vital part in the functioning of modern aircraft. Several different electric power systems have come into use. The earlier systems were direct current. Direct current systems are still being installed in aircraft. As this study so far has been practically confined to the three-phase 208/120-volt 400-cycle constant frequency system, the following discussion places emphasis on that system.

1. **Function of a Constant Frequency System**

A constant frequency a-c power system, in order to be most effective, must supply electric power to the load (utilization devices) at a definite voltage and at a definite frequency. The power system may therefore be considered as comprised of an alternating current generating system of one or more alternators and a transmission and distribution system consisting of buses, feeders and protective devices. The power system, however, must be coordinated with the characteristics of the utilization devices to perform under the following conditions:

a. Steady State

b. Transient

c. Short Time Continuous.

a. **Steady-State Operation**

During steady-state operation the electrical quantities are either constant or undergo very gradual changes. Operation under steady-state conditions should be automatically maintained at constant normal voltage and at constant normal frequency. Voltage regulators adjust the field excitation of the alternators automatically so as to maintain constant voltage. Speed
governors in the prime mover maintain constant normal (synchronous) speed thus maintaining constant frequency.

Generally alternating current power systems must furnish two kinds of power, namely, real power (watts or kilowatts) and reactive power (vars or kilovars). When two or more alternators are operating in parallel provision must be made to assure division of load current in accordance with the ratings. In the case of aircraft installations, to accomplish this the alternators divide automatically the real power in proportion to their respective ratings and divide automatically the reactive power in proportion to the respective ratings of the alternators. If the alternators all have the same ratings then both the real power and the reactive power should be shared equally. Division of real power is regulated by adjusting the mechanical output of the prime mover, usually by modifying the signal from the tachometer to the speed governor when the real power is divided improperly among the alternators. Division of reactive power is regulated by modifying the voltage to the voltage regulator when there is unequal division of reactive power.

The transmission and distribution system must transmit the real and reactive power to the load (utilization devices) with negligible voltage drop.

The required performance of the power system during steady-state operation may be summarized as follows:

1) Constant Normal Voltage at the Load
2) Constant Normal Frequency
3) Proper Division of Current Between Alternators

In order to analyze the performance under steady-state conditions
certain parameters of the alternators should be known. These include:

1) Potier Reactance
2) Direct-Axis Synchronous Reactance
3) Quadrature-Axis Synchronous Reactance
4) Amplification Factor of the Reactive Load Division Circuit
5) Amplification Factor of the Real Load Division Circuit.

b. Transient Conditions

Transient conditions may be defined as those which follow the sudden application or sudden removal of large loads including the application and clearing of short circuits. Under these conditions voltage dips and during most short circuits, voltage dips and voltage unbalances between phases cannot be avoided. The protective devices should remain inoperative during momentary or short time overloads but must operate on short circuits to remove the faulty part of the system. The system must remain stable during these conditions which means that if there are two or more alternators in parallel they should not fall out of synchronism. In order to assure stability the duration of voltage dips must be limited within certain values and short circuits should be cleared as quickly as possible. In order to analyze the behavior of the system under transient conditions the following characteristics of the alternators and transmission system should be known:

1) Transient Impedance
2) Negative-Sequence Impedance
3) Zero-Sequence Impedance
4) Voltage Regulator Response
5) Speed Regulator Response
c. Short-Time-Continuous Operation

Short-time-continuous operation may be considered as operation under severe overload. The duration and intensity of the overload is largely a matter of permissible temperature rises in the supply-system components and also a matter of permissible voltage reduction due to the regulation of the power system. In order to evaluate the performance of the system under these conditions, it is necessary to know the thermal characteristics as well as the steady-state electrical characteristics of the system.

2. Applications of Parameters

The parameters which govern the behavior of the system under steady-state operation and under transient conditions as outlined above can be obtained from tests. These parameters are highly useful in determining the performance of the system for a large variety of conditions. However, some discretion in the selection and application of these parameters is necessary. A case in point is the behavior of the system when an alternator is connected to a live bus with its voltage appreciably out of phase. In that case there will be swings of power in which the operation of the alternator alternates between generator action and motor action. These oscillations gradually die out. The reactance of the alternator during these swings is not constant even if the degree of magnetic saturation were constant. During these oscillations there will also be fluctuations in the field current so that we might say that the apparent reactance oscillates between values of saturated synchronous reactance and the transient reactance. Any analysis of synchronizing swings based upon the above parameters is at best rather approximate.

In spite of the fact that some of these parameters are not too
clear cut and, in a manner of speaking, have strings tied to them, it is almost impossible to effectively plan, analyze and coordinate a power system with its load without making use of these parameters. It would indeed be impractical to attempt mockups in order to determine experimentally the performance of a system under all possible conditions. However, mockups are practically indispensable for determining the behavior of a power system under certain conditions. As an example consider a fault which is intermittent due to vibration or perhaps other causes. Here a condition exists where the impedance of the fault may vary thru a range of values, perhaps a number of times in a second, all the way from a fraction of an ohm to infinity.

a. Saturated and Unsaturated Synchronous Reactance

1) Saturated Synchronous Reactance

The synchronous reactance of an alternator even with a fixed value of field current may vary over a wide range of values. A hypothetical case which might be considered extreme, although entirely possible, is that of an alternator delivering load at or above rated voltage with its field excitation sustained at ceiling level. Under this condition the 40-KVA alternators used in these studies have a saturated synchronous reactance of 0.15 per unit. Suppose that a three-phase short circuit of at least one second's duration is applied to this alternator with the field excitation maintained at ceiling level. During this short circuit, the flux linking the field winding of this alternator drops from a high value to such a low value, although the final field current is still at ceiling value, that the magnetic circuit is now unsaturated. For this final condition the synchronous reactance (unsaturated) has a value of 1.5 per unit or ten times its original
saturated value. Figure 3 shows the zero power-factor voltampere characteristic of the 40-kva alternator for ceiling excitation. This large variation of reactance from the saturated state to the unsaturated state is not peculiar to synchronous machines but is prevalent in any magnetic circuit which contains ferromagnetic materials. For steady-state analyses in which the synchronous reactance plays a part, it is usually necessary to take into account the degree of magnetic saturation. One widely used method is that of obtaining the air-gap voltage. In the case of a generator the Potier reactance drop is added vectorially to the terminal voltage to give the air-gap voltage. For motors the Potier reactance drop is subtracted from the terminal voltage to give the air-gap voltage. The degree of saturation is then determined by locating the air-gap voltage on the open-circuit saturation curve. There are several ways in which corrections can be applied for saturation each usually best suited for its particular application. Two examples of this application are the following methods for determining the voltage regulation of an alternator:

a) The General Method
b) The A.S.A. Method

Both of these methods are described in standard textbooks on a-c machinery.

2) Unsaturated Synchronous Reactance and Alternator Size

The use of the unsaturated synchronous reactance is generally restricted to applications where very approximate and usually very pessimistic results suffice. However, the value of unsaturated synchronous reactance affords some measure of the quality of performance of an alternator. Generally for equally sensitive automatic voltage regulators the alternator with the lower value of unsaturated synchronous reactance is more stable and
FIG. 3 - VOLT-AMPERE CHARACTERISTIC FOR CEILING EXCITATION AND ZERO P.F. CURRENT.
imposes less severe duty upon the voltage regulating system. The penalty for this superior performance is that for a given speed and KVA rating the alternator with the lower reactance is the larger in size making it heavier. The correlation between the size and unsaturated synchronous reactance of an alternator can be verified as follows. Consider an alternator with a certain length of air gap and a certain value of unsaturated synchronous reactance. If we double the length of the air gap in that alternator the unsaturated synchronous reactance drops to practically one-half the original value. This follows in any magnetic circuit with an air gap if the magnetic reluctance of the iron circuit is negligible. However, as soon as we double the length of the air gap in the alternator it is necessary to double the field ampere turns to obtain the same no load voltage. This means that the size of the field has to be increased. In order to prevent excessive temperature rises and at the same time to accommodate the larger field it is necessary to increase the overall size of the alternator.

b. Quadrature-Axis Synchronous Reactance

In the salient-pole alternator such as is used on aircraft the length of air gap is far from uniform. The air gap is shortest under the center of the field pole and very large halfway between field poles. The armature mmf then can be considered as being comprised of two parts, one reacting upon the direct axis, thru the center of the field poles, and the other component reacting upon the quadrature axis, thru the space midway between pole centers. From this division of the armature mmf we get the two synchronous reactances i.e. direct axis and quadrature axis. The two-reactance concept provides a tool by means of which the effect of saliency can be taken into account to a fair degree of approximation. The behavior
of a salient-pole alternator operating, with an open field, in parallel with other alternators can be readily analyzed particularly for light load conditions. In addition the two-reactance method makes possible the analysis of the transient behavior of synchronous machines to a degree which was not possible before this method was introduced.

c. Amplification Factor - Reactive Load Division

In order to determine the effectiveness of the reactive load division circuits of alternators, it is necessary to determine the change in excitation with unbalance of reactive load. In addition this amplification factor is required to predict the value of bus voltage and alternator loads in the event that the voltage regulator of one of several alternators operating in parallel loses its voltage sense. The use of such a factor in analytical methods provides an overall picture of the effects of unbalanced reactive load and malfunctioning of the excitation system of an alternator with much less expenditure of time and effort than is possible by test methods alone.

d. Amplification Factor - Real Load Division

The argument which has been applied to the reactive load division circuit and its effect upon excitation applies equally well to the real load division circuit and its effect upon speed governing. It applies also to the malfunctioning of the governor on the prime mover of one alternator operating in parallel with other alternators.

e. Transient Impedance

The transient impedance of an alternator is comprised of the a-c armature resistance and the transient reactance.
1) Transient Reactance

Under transient conditions there is still a further complication which is introduced by a transient in the field current. If we consider the same alternator which is delivering load at ceiling excitation, it will be appreciated that a relatively large amount of magnetic flux links the field winding. At the instant the short circuit is applied these flux linkages with the field must be the same as before short circuit. However, upon short circuit the armature current exerts a large opposing mmf upon the field and in order to maintain the flux linkages in the field winding, the field current rises possibly to several times its value before short circuit. Figure 8 shows oscillograms of a three-phase short circuit. The increase in field current and the a-c component in the field current are both evident in that figure. During short circuit both the field current and the armature current fall off in value until the field current attains the value it had before short circuit. The initial armature current during short circuit, if the current in the damper windings and pole faces of the field is negligible, is called the transient current. If the current in the damper windings or in the pole faces is appreciable, then the initial short-circuit armature current is even higher and is called the subtransient current. The transient current is said to be limited by the transient impedance and the subtransient current is said to be limited by the subtransient impedance.

It should be emphasized that the initial short-circuit current, i.e. the transient current or subtransient current whichever the case may be, is greater than the final steady-state current even if there were no magnetic saturation before short circuit, the increase over the final steady value of armature current being due to the initially increased field current.
Since circuit breakers and other fault clearing devices should clear faults as rapidly as possible, the interrupting duty for short-circuit currents is based on the initial value of the fault currents. Where large salient-pole alternators, with damper windings embedded in the field poles, are involved, the positive-sequence impedance is taken as the subtransient impedance. No appreciable subtransient effects were detected in these system studies with the 40-KVA, 400-cycle aircraft alternators and the transient impedance should therefore, be used for the positive-sequence impedance.

f. Negative-Sequence and Zero-Sequence Impedances

Under balanced three-phase conditions the only impedance we are concerned with is the positive-sequence impedance. Under normal conditions the positive-sequence impedance of an alternator is comprised of the a-c resistance of the armature (often neglected) and the synchronous reactance (direct-axis and quadrature-axis). However, under a balanced three-phase short circuit, for the initial conditions the transient impedance or subtransient impedance, whatever the case may be, is used. Under normal balanced operation, the armature mmf (if harmonics are neglected) rotates at synchronous speed in the same direction as the rotor and does not induce any current in the field circuit. However, under the initial conditions of a three-phase short circuit the current in at least two of the phases has a d-c component in addition to the a-c component. The a-c components in the three phases combine to produce an armature mmf which rotates at synchronous speed. The d-c components, however, in the three phases produce an armature mmf which is stationary and which induces a component of current in the field circuit which has fundamental frequency. This fundamental component in the field current produces a double-frequency (second harmonic),
component in the armature current. This second harmonic in the armature
current, usually negligible, exists only during the transient period. The
steady-state armature current is free of harmonics on a three-phase short
circuit.

The d-c component on short circuit will increase the current to
be interrupted by protective equipment. The effect of the d-c component on
the magnitude of the current to be interrupted depends upon the time con-
stant of the system and the operating time of the protective device.

1) Line-to-Line Short Circuit

If a short circuit is applied from line to line without involving
neutral, the fault current in two of the phases will be identical and the
fault current in the third phase will be zero. This single-phase armature
current in the two phases gives rise to a mmf which oscillates along a fixed
axis in the stator. This oscillating mmf can be split into two constant
equal components which rotate in opposite directions at synchronous speed.
We now have one of these components rotating in the direction in which the
field rotates. This component of mmf is stationary with respect to the field
and is said to be produced by the positive-sequence armature current.

a) Negative-Sequence Impedance

The component of mmf rotating in the opposite direction rotates
at twice synchronous speed relative to the field and is said to be produced
by the negative-sequence armature current. The alternator impedance assoc-
iated with the positive-sequence current is known as the positive-sequence
impedance. For short-circuit studies involving interrupting duty of circuit
breakers, the transient impedance or subtransient impedance, whichever the
case may be, is taken as the positive-sequence impedance. The alternator
impedance associated with the negative-sequence current is defined as the negative-sequence impedance. The negative-sequence impedance is different from the positive-sequence impedance of a rotating a-c machine. These considerations then show us that a line-to-line fault involves the positive-sequence impedance and the negative-sequence impedance of a power system.

b) Harmonics in Line-to-Line Fault Current

As the mmf, in the armature of an alternator, produced by the negative-sequence current rotates in a backward direction at synchronous speed, its speed relative to the rotor is twice synchronous speed. The fundamental component of the negative-sequence current then induces an alternating current in the alternator field which has twice fundamental frequency. This double-frequency component of field current then produces a field flux which oscillates at double frequency in the axis of the field poles. This oscillating field flux can also be split into two equal but oppositely rotating field fluxes each rotating at twice synchronous speed relative to the field. Taking into account the rotation of the field structure at synchronous speed in the forward direction, the forward rotating component of the oscillating field flux rotates at three times synchronous speed relative to the armature, and the backward rotating component of the oscillating field flux rotates at synchronous speed relative to the armature. The forward component of the oscillating field flux then generates a triple frequency voltage in the armature resulting in a triple frequency component in the fault current. Continuing this reasoning we find that the triple frequency component of the armature induces in the field winding a double frequency component of current and a component of four times fundamental frequency. From this it follows that theoretically the fault current and the voltage in
the unfaulted phase contains an infinite series of odd harmonics and the field current contains an infinite series of even harmonics. However, as a result of leakage inductances and the damping effects of damper windings, and paths for eddy currents in the field poles, the higher harmonics become negligible very rapidly. Figure 35 shows oscillographic records taken on a line-to-line fault. There is a pronounced second harmonic in the field current and there are odd harmonics in the armature voltages. These harmonics in the voltage of the unfaulted phase, in the absence of damper windings in the faces of the field poles, have been known to rise to destructive values as a result of series resonance with the capacitance of the system, in the case of hydroelectric alternators connected to long transmission lines.

2) **Line-to-Ground Fault**

In the case of the line-to-line fault it was shown that the armature develops two constant equal but oppositely rotating mmfs. Each of these mmfs is produced by an equivalent balanced three-phase system of currents. One of these systems of current has positive phase sequence and the other has currents of the same magnitude as the positive-sequence currents but of negative phase sequence. Further the phase relationships of these systems of currents is such that the positive-sequence component in the unfaulted phase is equal and opposite to the negative-sequence component of current in the unfaulted phase. Neither of these balanced system of currents can give rise to neutral current. However, in the case of the single line-to-ground fault the armature produces an oscillating mmf but there is also neutral current. This oscillating mmf in only one phase cannot be explained on the basis of two sets of three-phase currents of opposite phase sequences because
there is neutral current. If, however, we consider this mmf as being produced by three systems of balanced current in all three phases we can take neutral current into account simply by adding to the positive-sequence currents and negative-sequence currents three equal single-phase currents, one for each phase, known as zero-sequence current. Further, the phase relationships and magnitudes are such that when the three different sequence components of current are combined, the resultant current in the unfaulted phases is zero. The effect of the positive-sequence and negative-sequence components of current are very much the same as in the case of the line-to-line fault. The effect of the negative-sequence component of armature current is such as to produce a series of even harmonics in the field current which in turn produces a series of odd harmonics in the armature current. These effects are evident in Fig. 32 which shows an oscillographic record of a single-line-to-ground fault.

a) **Zero-Sequence Impedance**

First of all let it be emphasized that in order for zero-sequence current to exist there must be a neutral path. In aircraft alternators this is practically always the case as the neutrals of the alternators are usually grounded. The resultant mmf produced by the zero-sequence current as far as the rotor or field structure is concerned is zero as we have three equal mmfs in time phase with each other but directed along magnetic axes which are displaced 120 electrical degrees from each other in space. The zero-sequence impedance is the impedance which is associated with the zero-sequence current and includes the effect of the neutral path external to the alternator.
3. **Analytical vs. Experimental Methods**

In order to make a systematic study it is necessary to have a thorough understanding of the parameters involved. Their magnitudes and limitations must be known. With the proper application of such knowledge reliable results can be obtained quickly by means of analytical methods which otherwise involve excessively time consuming and costly staged tests on a power system. In analytical short-circuit studies, for example, use is made of the sequence impedances. The sequence impedances of rotating machines are usually obtained from tests but can also be predicted from machine design data. In studies of large and complex power systems, a-c network calculators are used to a large extent. In such calculators networks adjusted to the proper values of the various sequence impedances are connected so as to simulate different types of faults. Such methods i.e. those which are purely analytical and those which make use of the network calculator are particularly advantageous if the parameters may be held constant.

It is true that expressions have been developed which theoretically take into account fault currents expressed as simple exponential functions of time. However, such analyses can become tedious and they also suffer from such inaccuracies as not taking changes of magnetic saturation into account. In such cases staged tests on the actual power system or its mockup yield reliable results at a saving in time. Staged tests on large power systems are not always practical because of the tremendous amounts of power involved. However, in the case of aircraft power systems, mockups using conventional aircraft alternators and conventional conductors, buses, circuit breakers, fuses and relays are convenient, economical and reliable for obtaining information which cannot be obtained as readily or as exactly by purely analytical
means. Such mockups backed by analytical studies are more economical than would be the installation and use of a conventional a-c network calculator for aircraft power system studies.

The chief limitation of staged tests on full scale system mockups is the lack of flexibility in adjusting characteristics of the system components. At present analytical methods must be used to determine effects of changing system parameters.

4. **Extent of Present Study**

The study described in this section covers the determination of those parameters which govern the system performance under

a. Steady-State Conditions
b. Transient Conditions.

The parameters which determine the behavior of the system under short-time-continuous conditions are usually determined in machine acceptance tests. The transient conditions in a broad sense would include operation during short circuit and alternator blocked rotor in addition to synchronizing disturbances and overvoltages.

The analysis of the voltage regulator, speed regulator, and equalizing circuits deals with the steady-state response. The transient response can be determined by treating these systems as servomechanisms. This includes considering the hydraulic action of the constant speed device, mechanical gearing, and electrical circuits as a whole system.
B. Alternator Characteristics

The experimental investigation in these system studies were conducted at Wright-Patterson Air Force Base, Dayton, Ohio in building T-47, on a 400 cycle a-c system which included wiring, alternators, controls, constant speed drives, and drive stands. Two 400 KVA alternators designated as alternator A and alternator D driven by constant speed drives were used in these tests. In addition use was made of two similar alternators connected to drive stands in building T-47 without intervening constant speed transmissions. The name plate data of alternator D, its associated constant speed drive, and drive stand appear in Appendix III-A. Figure 4 shows the alternator, constant speed drive, and a part of the drive stand. The control panel as well as a general view showing the specific items mentioned above are shown in Fig. 5.

1. Steady State-Balanced Operation

The alternator may be considered an ideal voltage source feeding the load through a variable impedance. Under balanced conditions only one phase of the alternator need be considered, and the equation for the terminal voltage of the alternator is

\[ V = E - I Z_d \]  

where:

- \( V \) = terminal voltage per phase,
- \( E \) = generated voltage per phase,
- \( I \) = armature current per phase,
- \( Z_d \) = synchronous impedance per phase.

Figure 6 shows the equivalent circuit and vector diagram of an alternator.

In order to obtain the steady state parameters, tests were made on alternator D to determine the following:
FIG. 6 - EQUIVALENT CIRCUIT AND VECTOR DIAGRAM OF AN ALTERNATOR
1) No load saturation curve  
2) Three-phase short-circuit saturation curve  
3) Zero power-factor saturation curve.

a. No Load Saturation Curve

In order to determine this characteristic, alternator D was driven without load at synchronous speed (6000 rpm), with its field excited by supplying the exciter field from a battery through an adjustable resistance. A current shunt was placed in series with one of the leads to the alternator field slip rings in order to measure the alternator field current. The terminal voltage of alternator D for various values of alternator field current was measured. The results are given in Table VIII. The per unit values for a base field current of 15.8 amps and a base voltage of 120 volts are also listed in Table VIII. It is common practice to take that value of field current as a base i.e. one per unit which corresponds to rated voltage or one per unit voltage on the air-gap line. This makes the air-gap lines of all alternators identical when plotted in per unit.

Figure 7 shows the no load saturation curve which was plotted from the results given in Table VIII. Also shown is the air-gap line which would be the relationship between the terminal voltage and the field current, if the iron had negligible reluctance.

b. Three-Phase Short-Circuit Saturation Curve

With alternator D at a standstill, the three phases were short circuited at the terminals, one phase through a current transformer in order to measure the armature short-circuit current. The alternator field current was measured as in the no load saturation test. In the short-circuit test, alternator D was separately excited and driven roughly at synchronous speed.
### Table VIII

NO LOAD TEST DATA ON ALTERNATOR-D  
(Name Plate Data Listed in Appendix III-A)  

Synchronous Speed = 6000 RPM

<table>
<thead>
<tr>
<th>Line-to-Neutral Voltage</th>
<th>Alternator Field Current</th>
</tr>
</thead>
<tbody>
<tr>
<td>Volts</td>
<td>Per Unit</td>
</tr>
<tr>
<td>24.5</td>
<td>0.204</td>
</tr>
<tr>
<td>41.5</td>
<td>0.237</td>
</tr>
<tr>
<td>45.5</td>
<td>0.379</td>
</tr>
<tr>
<td>49.5</td>
<td>0.412</td>
</tr>
<tr>
<td>52.5</td>
<td>0.447</td>
</tr>
<tr>
<td>57.5</td>
<td>0.479</td>
</tr>
<tr>
<td>61.0</td>
<td>0.508</td>
</tr>
<tr>
<td>65.0</td>
<td>0.546</td>
</tr>
<tr>
<td>72.8</td>
<td>0.606</td>
</tr>
<tr>
<td>78.0</td>
<td>0.650</td>
</tr>
<tr>
<td>84.0</td>
<td>0.706</td>
</tr>
<tr>
<td>91.3</td>
<td>0.760</td>
</tr>
<tr>
<td>98.5</td>
<td>0.820</td>
</tr>
<tr>
<td>106.0</td>
<td>0.883</td>
</tr>
<tr>
<td>111.4</td>
<td>0.930</td>
</tr>
<tr>
<td>113.0</td>
<td>0.914</td>
</tr>
<tr>
<td>115.5</td>
<td>0.962</td>
</tr>
<tr>
<td>116.0</td>
<td>0.966</td>
</tr>
<tr>
<td>118.0</td>
<td>0.984</td>
</tr>
<tr>
<td>119.6</td>
<td>0.995</td>
</tr>
<tr>
<td>121.4</td>
<td>1.010</td>
</tr>
<tr>
<td>123.0</td>
<td>1.025</td>
</tr>
<tr>
<td>125.0</td>
<td>1.045</td>
</tr>
<tr>
<td>127.0</td>
<td>1.060</td>
</tr>
<tr>
<td>129.2</td>
<td>1.075</td>
</tr>
<tr>
<td>133.0</td>
<td>1.111</td>
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<tr>
<td>137.2</td>
<td>1.113</td>
</tr>
<tr>
<td>140.0</td>
<td>1.165</td>
</tr>
<tr>
<td>149.2</td>
<td>1.212</td>
</tr>
</tbody>
</table>
FIG. 7 - ALTERNATOR CHARACTERISTICS
For the short-circuit test, the effect of speed upon the short-circuit current is not critical, since both the induced voltage and reactance are directly proportional to the speed. At speeds as low as 1/4 synchronous speed, the effect of armature resistance on the short-circuit current is negligible.

Readings of armature short-circuit current and field current were taken. The measured values are shown in Table IX and the results are shown graphically in Fig. 7. Rated armature current of 111.1 amps was used as the base armature current or one per unit. In the short-circuit test the armature mmf is practically in direct opposition to the field mmf causing the resultant air-gap flux to be too low to produce saturation.

c. Synchronous Reactance

For low values of field current, the no load saturation curve is a linear characteristic since the iron of the alternator is unsaturated. The unsaturated synchronous impedance is the ratio of the voltage on the air-gap line to the armature current on a three-phase short-circuit for a given value of field current. From Fig. 7 the unsaturated synchronous reactance, \( X_{du} = \frac{Z_{du}}{I_{dul}} = 1.50 \) per unit.

For higher values of field current, the synchronous reactance \( X_d \) does not remain constant because of the effect of saturation. The variation in the synchronous reactance \( X_d \) is indicated on Fig. 7 by the dashed line. This reactance is obtained from the no load saturation curve and the short-circuit saturation curve.

d. Short-Circuit Ratio

The short-circuit ratio is defined by the following quotient:

\[
SCR = \frac{\text{Field current for rated open-circuit voltage}}{\text{Field current for rated short-circuit current}}
\]

If there were no saturation, the short-circuit ratio would be the reciprocal.
Table IX
THREE-PHASE SHORT-CIRCUIT TEST DATA ON ALTERNATOR D
(Name Plate Data Listed in Appendix III-A)

<table>
<thead>
<tr>
<th>Armature Current</th>
<th>Alternator Field Current</th>
</tr>
</thead>
<tbody>
<tr>
<td>Amperes</td>
<td>Per Unit</td>
</tr>
<tr>
<td>35.6</td>
<td>0.321</td>
</tr>
<tr>
<td>44.2</td>
<td>0.397</td>
</tr>
<tr>
<td>53.0</td>
<td>0.477</td>
</tr>
<tr>
<td>70.6</td>
<td>0.635</td>
</tr>
<tr>
<td>100.0</td>
<td>0.90</td>
</tr>
</tbody>
</table>

Table X
ZERO POWER-FACTOR TEST DATA ON ALTERNATOR D

<table>
<thead>
<tr>
<th>Phase Voltage</th>
<th>Field Current</th>
<th>Armature Current</th>
</tr>
</thead>
<tbody>
<tr>
<td>Volts</td>
<td>Amperes</td>
<td>Amperes</td>
</tr>
<tr>
<td>Volts Per Unit</td>
<td>Amperes</td>
<td>Per Unit</td>
</tr>
<tr>
<td>126.5</td>
<td>1.05</td>
<td>49.7</td>
</tr>
</tbody>
</table>
of the unsaturated synchronous reactance expressed in per unit. The greater the short-circuit ratio of an alternator, the greater is its steady-state stability. However for a given speed and KVA rating, the weight and size of the alternator increase with increasing short-circuit ratio.

The short-circuit ratio of alternator D was found to be 0.90. According to Crary,\(^1\) where dependence is placed on manual control of excitation, a short-circuit ratio of around unity is used. In Europe, with the use of automatic voltage regulators, short-circuit ratios as low as 0.4 are not uncommon. A smaller and lighter alternator can therefore be used if greater responsibility is placed on the voltage regulator. Although detailed analyses of stability for the aircraft a-c system, were not made for the present study, it appears that machines with a lower short-circuit ratio could be tolerated.

\(\text{e. Zero Power-Factor Saturation Curve}\)

Under normal operating conditions the armature current reacts upon a magnetic circuit in which there is some magnetic saturation. As mentioned in the foregoing the iron of a conventional alternator is unsaturated in the short-circuit test. Therefore the performance, of the alternator under load, cannot be predicted with reasonable precision on the basis of the \(X_d\) versus \(I_f\) characteristic in Fig. 7. More reliable data can be obtained by making use of the zero power-factor saturation curve which is obtained under conditions in which the armature current is present when the iron is saturated.

In order to obtain the zero power-factor characteristic, the no

load saturation curve is required and in addition two values of field current, one to produce a certain value of three-phase short-circuit current and the other the same value of armature current for zero power-factor at rated terminal voltage or above.

In the zero power-factor tests made at Wright Field, alternator D was loaded by using alternator A as an underexcited, unloaded synchronous motor in order to obtain a zero power-factor load at about rated voltage. The test data are shown in Table X. The point corresponding to 1.05 per unit terminal voltage and 0.79 per unit current is indicated in Fig. 7.

The other point for the zero power-factor characteristic corresponds to zero terminal voltage and 0.79 per unit armature current. This point was obtained from the three-phase short-circuit test on alternator D. Since the impedance of the alternator is largely reactive, a short circuit at the alternator terminals involves a zero power-factor current. This point is shown on Fig. 7 and corresponds to an armature current of 0.79 per unit at zero terminal voltage.

From these two points the Potier triangle may be constructed and thus the entire zero power-factor saturation curve may be determined by moving the Potier triangle parallel to itself along the no load saturation curve. Because Potier triangles for different values of armature current for the same alternator are similar, the zero power-factor characteristic can be drawn for any value of armature current once the Potier reactance and armature demagnetizing factor are found.

f. Potier Reactance and Armature Demagnetizing Factor

In order to determine the Potier reactance and the demagnetizing effect of the armature current, the Potier triangle is constructed by using
the two zero power-factor points and the slope of the air-gap line as shown in Fig. 7. The vertical side of the Potier triangle represents the Potier reactance drop; and the horizontal side the demagnetizing effect of the armature current. Since the triangle was constructed for an armature current of 0.79 per unit the Potier reactance \( \frac{0.084}{0.79} = 0.106 \) per unit and the armature demagnetizing factor \( \frac{1.13}{0.79} = 1.43 \) per unit. For any other armature current, the Potier triangle may be constructed and the corresponding zero power-factor curve drawn.

g. Direct-Axis and Quadrature-Axis Synchronous Reactance

For a salient-pole alternator, the air-gap is not uniform and it is convenient to use quadrature-axis and direct-axis quantities. Therefore the voltage equation for a salient-pole alternator, neglecting armature resistance, is

\[
V = E - j X_d I_d - j X_q I_q
\]

where:

- \( V \) = terminal voltage per phase,
- \( E \) = generated voltage per phase,
- \( X_d \) = direct-axis synchronous reactance per phase,
- \( X_q \) = quadrature-axis synchronous reactance per phase,
- \( I_d \) = direct-axis armature current per phase,
- \( I_q \) = quadrature-axis armature current per phase.

In order to determine the quadrature-axis synchronous reactance, a slip test was made on alternator D.

1) Slip Test

The slip test was performed by using alternator D and one of the other alternators which was coupled directly to its drive stand. The other
alternator which was mounted on drive stand 4 was used as a three-phase source of voltage of rated frequency. This alternator was separately excited and reduced voltage of about 0.47 per unit was applied to the armature of alternator D. With the field winding open-circuited, alternator D was driven slightly above synchronous speed. In this way the field poles of alternator D slipped past the magnetic poles which were produced by the armature current and which were revolving at synchronous speed. As the two sets of poles change their relative position the reactance varies and the armature current and armature voltage oscillate. When the voltage is a maximum and the current is a minimum, the poles are lined up and the ratio of voltage to current gives the direct-axis reactance. Similarly the ratio of the minimum voltage and the maximum current gives the quadrature-axis reactance. From the slip test the following values were obtained:

\[ X_q = 0.91 \text{ per unit for the quadrature-axis synchronous reactance} \]

\[ X_d = 0.97 \text{ per unit for the direct-axis synchronous reactance}. \]

The speed of alternator D could not be maintained at a constant value slightly above synchronous speed because of the action of the free-wheeling clutch. The free-wheeling clutch is a safety device incorporated in the constant speed drive which disengages the alternator from the constant speed drive when the alternator is operating as a motor.

The field poles would tend to line up in the direct-axis so as to keep the electrical energy at a minimum. Therefore, the rotor of alternator D did not slip slowly past the magnetic poles, but had a pulsating motion. This resulted in a relatively high slip speed and currents were induced in the damper windings. Since the damper windings are not interconnected, the induced currents only circulated in the direct-axis. Therefore, the value
for $X_d$ obtained from this test would be low, but that for $X_q$ would not be materially affected. The direct-axis reactance is already available from other tests and would correspond to the unsaturated value of 1.5 per unit since approximately half-rated voltage was applied. The damper currents would account at least in part for the lower value of $X_d$ obtained on the slip test.

2) Transient-Unbalanced Operation

In order to analyze unbalanced operation of the alternators, use is made of the positive, negative, and zero sequence quantities. The positive-sequence impedance is usually assumed to be equal to the transient impedance and therefore is also used in calculations involving balanced conditions where the initial quantities are to be determined. Because of this fact, an analysis of oscillographic records of the armature currents immediately after the occurrence of a three-phase short-circuit provides a method of determining the positive-sequence impedance.

a. Positive-Sequence Impedance

The initial value of the short-circuit current in an alternator which has damper windings is limited by the subtransient impedance. This high value of current rapidly drops to a value which is limited by the transient impedance. This transient current in turn decays eventually to the steady-state value determined by the synchronous impedance.

In order to determine the transient impedance of alternator $A$, a three-phase line-to-line short circuit was suddenly applied while the alternator was running unloaded and oscillographic records were taken. Since the fault was made at the bus, the transient impedance obtained from the oscillographic records is for the alternator and the cable to the bus. Oscillographic traces of the three phase voltages measured at the bus, three line currents,
and alternator field current were recorded. Figures 8 and 9 are reproductions of two different oscillographic records obtained on three-phase faults.

Since the rate of decay of the transient component of fault current approximates an exponential function, a graph of this transient component on semi-log paper as a function of time approximates a straight line. To determine the transient impedance the envelopes of the phase currents and arbitrary spaced time markers are sketched in on the oscillographic records. Since the armature currents are offset due to a d-c transient, half of the peak to peak value is taken as the basis for the total a-c short-circuit current. From this current, the steady-state short-circuit current is subtracted. The steady-state value of short-circuit current would be the current when the field current drops to the value it had before the fault. As may be seen by the oscillograms the field current then increases to a high value through the action of the voltage regulator. The result of subtracting the steady-state value from the short-circuit current leaves the transient component. These values are listed in Table XI and are plotted as a function of time in Fig. 10 and Fig. 11.

The intercept on the vertical axis of a straight line drawn through the points gives the peak value of the initial transient current minus that of the steady-state current. The transient impedance is then obtained by dividing the peak value of the rated phase voltage by the initial transient current. The transient reactance of the alternator and the cable to the bus is thus found to be 0.15 per unit. If the positive-sequence reactance of the cable is taken to be 0.005 per unit\(^1\), then the transient reactance

\(^1\text{Army Air Force Report, "A-C System Fuse Coordination Study" (Dec. 1945), p.23.}\)
FIG. 8 THREE-PHASE SHORT CIRCUIT ON ONE ALTERNATOR.
Fig. 9 Three-Phase Short Circuit On One Alternator
Table XI
THREE-PHASE SHORT-CIRCUIT DATA ON ALTERNATOR-A
(Name Plate Data Listed in Appendix III-A)

Tabulation Refers to Fig. 8 and Fig. 10

<table>
<thead>
<tr>
<th>Time (Ms.)</th>
<th>$i_a$ (Amperes)</th>
<th>$i_a - 160^*$ (Amperes)</th>
<th>$i_b$ (Amperes)</th>
<th>$i_b - 160$ (Amperes)</th>
<th>$i_c$ (Amperes)</th>
<th>$i_c - 160$ (Amperes)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.5</td>
<td>780</td>
<td>620</td>
<td>700</td>
<td>540</td>
<td>780</td>
<td>620</td>
</tr>
<tr>
<td>5.0</td>
<td>550</td>
<td>390</td>
<td>510</td>
<td>350</td>
<td>540</td>
<td>380</td>
</tr>
<tr>
<td>7.5</td>
<td>430</td>
<td>270</td>
<td>390</td>
<td>230</td>
<td>390</td>
<td>230</td>
</tr>
<tr>
<td>10.0</td>
<td>350</td>
<td>190</td>
<td>310</td>
<td>150</td>
<td>310</td>
<td>150</td>
</tr>
<tr>
<td>12.5</td>
<td>290</td>
<td>130</td>
<td>270</td>
<td>110</td>
<td>260</td>
<td>100</td>
</tr>
</tbody>
</table>

Tabulation Refers to Fig. 9 and Fig. 11

<table>
<thead>
<tr>
<th>Time (Ms.)</th>
<th>$i_a$ (Amperes)</th>
<th>$i_a - 160$ (Amperes)</th>
<th>$i_b$ (Amperes)</th>
<th>$i_b - 160$ (Amperes)</th>
<th>$i_c$ (Amperes)</th>
<th>$i_c - 160$ (Amperes)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.5</td>
<td>780</td>
<td>620</td>
<td>730</td>
<td>570</td>
<td>760</td>
<td>600</td>
</tr>
<tr>
<td>5.0</td>
<td>560</td>
<td>400</td>
<td>510</td>
<td>350</td>
<td>560</td>
<td>400</td>
</tr>
<tr>
<td>7.5</td>
<td>440</td>
<td>280</td>
<td>390</td>
<td>230</td>
<td>440</td>
<td>280</td>
</tr>
<tr>
<td>10.0</td>
<td>340</td>
<td>180</td>
<td>310</td>
<td>150</td>
<td>330</td>
<td>170</td>
</tr>
<tr>
<td>12.5</td>
<td>260</td>
<td>100</td>
<td>230</td>
<td>70</td>
<td>260</td>
<td>100</td>
</tr>
</tbody>
</table>

*Note: Steady-state short-circuit current = 160 amperes before regulator action increases field excitation.
FIG. 10 - DETERMINATION OF TRANSIENT REACTANCE. CORRESPONDS TO OSCILLOGRAPH (FIG. 8)
FIG. 11 - DETERMINATION OF TRANSIENT REACTANCE.
CORRESPONDS TO OSCILLOGRAPH (FIG. 9)
of the alternator = 0.146 per unit. The d-c resistance of the alternator and
the cable has been used for the positive-sequence resistance.

b. Negative-Sequence Impedance

Negative-sequence currents flow during line-to-line, double line-
to-ground, and single-line-to-ground faults. Two different methods were
used to determine the negative-sequence impedance of alternator D.

1) Average of $Z_{d''}$ and $Z_{q''}$

A single-phase voltage was applied across two phases of alternator
D. Alternator A, separately excited, was used as the source for the voltage.
With the field of alternator D short-circuited, the rotor was slowly rotated.
As the magnetic axes of the field poles were made to approach the magnetic
axes of the armature, the armature current increased and the armature voltage
decreased reaching maximum and minimum values respectively when the two sets
of magnetic axes coincided. The ratio of one half the minimum voltage to
the maximum current yields the magnitude of the direct-axis subtransient
impedance $Z_{d''}$. This magnitude was found to be 0.146 per unit. Under this
single-phase condition when the magnetic axes of the armature and field poles
coincide the impedance is a minimum because of the current induced in the
field circuit and damper windings. When the magnetic axes of the field poles
lie half way between the magnetic axes of the armature, the armature current
is a minimum and the voltage is a maximum. From the values of maximum volt-
age and minimum current the magnitude of the quadrature-axis subtransient
reactance was found to be $Z_{q''} = 0.538$ per unit. Under this condition the in-
duced field current is negligible. The negative-sequence impedance neglect-
ing inequalities in impedance angles is the average of $Z_{q''}$ and $Z_{d''}$ and so
$Z_2 = 0.342$ per unit. The readings and calculations are shown in Table XII.
### Table XII

**SUTRANSIENT IMPEDANCE AND NEGATIVE-SEQUENCE IMPEDANCE TEST DATA**  
(Name Plate Data Listed in Appendix III-A)

<table>
<thead>
<tr>
<th>Line-to-Line Voltage Volts</th>
<th>Current Amperes</th>
<th>Impedance Ohms</th>
<th>P.U.</th>
</tr>
</thead>
<tbody>
<tr>
<td>27.7</td>
<td>48</td>
<td>0.578</td>
<td>0.535</td>
</tr>
<tr>
<td>28.1</td>
<td>48</td>
<td>0.586</td>
<td>0.542</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Average Value: Z_q&quot; = 0.538</td>
<td></td>
</tr>
<tr>
<td>16.5</td>
<td>85</td>
<td>0.194</td>
<td>0.180</td>
</tr>
<tr>
<td>12.0</td>
<td>100</td>
<td>0.120</td>
<td>0.111</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Average Value: Z_d&quot; = 0.146</td>
<td></td>
</tr>
</tbody>
</table>

Negative-Sequence Impedance: \[ Z_2 \approx \frac{Z_d" + Z_q"}{2} = 0.342 \text{ P.U.} \]
2) **Wattmeter Method**

Another method for determining the negative-sequence impedance of an alternator is perhaps more realistic than the foregoing method in that the alternator is operated under line-to-line fault condition. Further, by means of the wattmeter measurement, the negative-sequence resistance and negative-sequence reactance can be determined in addition to the negative-sequence impedance.

Two phases of alternator D were short-circuited at the terminals as shown in Fig. 12a. A voltmeter and the potential coil of a wattmeter were connected from the un-faulted phase to one of the short-circuited phases. The current coil of the wattmeter and an ammeter were connected to the secondary of a current transformer which in turn was placed in one of the short-circuited phases. With alternator D running at synchronous speed and the field separately excited, reduced excitation was applied. For different values of excitation, readings of voltage, current, and power were taken, and the results appear in Table XIII. From this wattmeter method, the negative-sequence impedance is \( Z_2 = (0.137 + j 0.217) \) per unit for an armature current of \( 1\frac{3}{8} \) amperes as indicated in Table XIII.

c. **Zero-Sequence Impedance**

Since the neutrals of aircraft alternators are grounded, a path for zero-sequence current is provided. To determine the zero-sequence impedance, the three phases of alternator D were connected in series as shown in Fig. 12b. A single-phase voltage obtained from alternator A was applied across the series combination. With alternator D running near synchronous speed and with the alternator field short-circuited, readings of the applied voltage and the current through the series combination were taken. The
FIG. 12a - WATTMETER METHOD OF DETERMINING THE NEGATIVE-SEQUENCE IMPEDANCE.

FIG. 12b - CONNECTIONS FOR MEASURING THE ZERO-SEQUENCE IMPEDANCE.
Table XIII

NEGATIVE-SEQUENCE IMPEDANCE OF ALTERNATOR-D FROM WATTMETER METHOD
(Name Plate Data Listed in Appendix III-A)

<table>
<thead>
<tr>
<th>E Volts</th>
<th>I Amperes</th>
<th>W Watts</th>
<th>( Z_2 = \frac{E}{\sqrt{3}I} )</th>
<th>( X_2 = \frac{Z_2 W}{EI} )</th>
<th>( R_2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>26.5</td>
<td>52</td>
<td>1240</td>
<td>0.272</td>
<td>0.245</td>
<td>0.119</td>
</tr>
<tr>
<td>46.0</td>
<td>94</td>
<td>3760</td>
<td>0.262</td>
<td>0.228</td>
<td>0.129</td>
</tr>
<tr>
<td>71.0</td>
<td>148</td>
<td>8920</td>
<td>0.257</td>
<td>0.217</td>
<td>0.137</td>
</tr>
</tbody>
</table>

Table XIV

ZERO-SEQUENCE IMPEDANCE OF ALTERNATOR-D

<table>
<thead>
<tr>
<th>E Volts</th>
<th>P.U.</th>
<th>I Amperes</th>
<th>P.U.</th>
<th>( Z_0 ) P.U.</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.13</td>
<td>0.0345</td>
<td>78.8</td>
<td>0.709</td>
<td>0.0487</td>
</tr>
<tr>
<td>4.60</td>
<td>0.0383</td>
<td>85.4</td>
<td>0.768</td>
<td>0.0498</td>
</tr>
<tr>
<td>4.50</td>
<td>0.0375</td>
<td>83.0</td>
<td>0.748</td>
<td>0.0502</td>
</tr>
</tbody>
</table>

Average Value: \( Z_0 = 0.0496 \)
results are shown in Table XIV. Generally the zero-sequence resistance is taken equal to the positive-sequence resistance of the alternator hence the zero-sequence impedance is \( Z_o = (0.02 \angle 0.043) \) per unit.

3. Discussion

The various values of reactance, short-circuit ratio and demagnetization factor determined in this study are listed in Table XV, along with values of reactance submitted by the Westinghouse Electric Corporation for the same type alternator. The values of unsaturated synchronous reactance \( X_{du} \) are in exact agreement. This is rather to be expected as the procedure for determining the unsaturated synchronous reactance is rather simple and straightforward. There is some disagreement between the values obtained for the transient, subtransient, and sequence reactances. With the exception of the unsaturated synchronous reactance, \( X_{du} \), the values obtained in this study are consistently lower than the values for the corresponding quantities submitted by Westinghouse. Exact agreement between these values however need not invalidate one set of results or the other as there are such variables as degrees of magnetic saturation, inherent inaccuracies in obtaining and interpreting oscillographic records, which are not duplicated exactly by different observers or necessarily by the same observer making tests at different times on two separate alternators of the same design and manufacture. There are however some approximate relationships which serve as rough checks on the exactness of some of these reactances.

a. Potier Reactance and Transient Reactance

Table XV lists a value of 0.106 per unit for the Potier reactance. The Potier reactance, also known as the leakage reactance, of a synchronous machine results from two leakage fluxes, namely, armature leakage flux and
Table XV

ALTERNATOR PARAMETERS

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Symbol</th>
<th>Value</th>
<th>&quot;W&quot;</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unsaturated Synchronous Reactance</td>
<td>$X_{du}$</td>
<td>1.50</td>
<td>1.50</td>
</tr>
<tr>
<td>Short-Circuit Ratio</td>
<td>SCR</td>
<td>0.90</td>
<td></td>
</tr>
<tr>
<td>Potier Reactance</td>
<td>$X_p$</td>
<td>0.106</td>
<td></td>
</tr>
<tr>
<td>Armature Demagnetizing Factor</td>
<td></td>
<td></td>
<td>1.43</td>
</tr>
<tr>
<td>Quadrature-Axis Synchronous Reactance</td>
<td>$X_q$</td>
<td>0.91</td>
<td></td>
</tr>
<tr>
<td>Direct-Axis Transient Reactance</td>
<td>$X_{d'}$</td>
<td>0.145</td>
<td>0.21</td>
</tr>
<tr>
<td>Direct-Axis Subtransient Reactance</td>
<td>$X_{d''}$</td>
<td>0.146</td>
<td>0.214</td>
</tr>
<tr>
<td>Quadrature-Axis Subtransient Reactance</td>
<td>$X_{q''}$</td>
<td>0.538</td>
<td>0.649</td>
</tr>
<tr>
<td>Negative-Sequence Reactance (average of $X_{d''}$ and $X_{q''}$)</td>
<td>$X_2$</td>
<td>0.342</td>
<td>0.432</td>
</tr>
<tr>
<td>Negative-Sequence Impedance (148 amp on Wattmeter Method)</td>
<td>$Z_2$</td>
<td>$0.137 + j0.217$</td>
<td></td>
</tr>
<tr>
<td>Zero-Sequence Reactance</td>
<td>$X_0$</td>
<td>0.043</td>
<td>0.084</td>
</tr>
</tbody>
</table>

*These values were submitted in a letter dated July 23, 1951 from Mr. R. H. Keith of Westinghouse Electric Corporation to Mr. L. W. Matsch of Armour Research Foundation.
field leakage flux. If it were not for the increased field leakage flux due to the transient increase in field current during short circuit, the transient reactance would have the same value as the Potier reactance. This increase in field leakage flux causes the transient reactance to be somewhat higher than the Potier reactance. Sterling Beckwith\textsuperscript{1} gives the following empirical relationship between Potier reactance and transient reactance: \( X_d' = 1.25 X_p \).

On the basis of a Potier reactance of 0.11 per unit the transient reactance should be approximately 0.14 per unit as compared with our value of 0.145 per unit. This relationship however is not hard and fast and is therefore not to be taken too literally. However, it does serve as a rough check.

b. **Short-Circuit Currents, Observed and Calculated**

Table XXIV shows a comparison of values of short-circuit current obtained by short-circuit tests with values of current computed from the parameters determined in this study. These values are in fair agreement as the calculated value for single-line-to-ground fault current is 673 amperes compared with a measured value of 631 amperes. The manufacturer's data would give a calculated value of 461 amperes not including the impedance of the line and neglecting the resistance of the alternator and line entirely. In addition to these discrepancies between different observers, different methods used by the same observer lead to different values of negative-sequence reactance. In our studies the negative-sequence reactance, computed on the basis of the average of \( X_d'' \) and \( X_q'' \), was 0.342 per unit and the value obtained with the wattmeter test was found to be 0.217 per unit for one value of field current; for lower values of field current the negative-

\textsuperscript{1}Sterling Beckwith, "Approximating Potier Reactance", AIEE Transactions (July 1937), p.813.
sequence reactance had a greater value indicating that the negative-sequence reactance decreases with magnetic saturation. These variations in the results obtained by different observers and the variations in results obtained by the same observer using different methods indicate a need for establishing test procedures which will lead to more consistent and more useful data.

C. **Regulators**

In order for the electric power system to deliver power to the utilization devices at the proper voltage and proper frequency, it is necessary to make adjustments in the field excitation of the alternators and in the output of the prime movers. In aircraft both of these types of adjustments should be made automatically. It is common practice to adjust field excitation automatically by means of a voltage regulator and to adjust the prime mover output automatically by means of a speed governor.

1. **Voltage Regulator**

The function of a voltage regulator is to maintain nearly constant terminal voltage on a generator with varying loads. The method of voltage regulation employed on the present a-c system is that of controlling exciter output by use of a variable resistance in series with the shunt field of the exciter. This resistor is a carbon stack, the resistance of which is controlled by a solenoid acting against a spring. The change in pressure on the carbon discs changes the resistance. In order that the regulator be complete and self operating, a method is required whereby under normal conditions a potential proportional to the terminal voltage of the alternator is applied to the solenoid such that a change in terminal voltage will cause a corresponding change in exciter field circuit resistance which will return
the terminal voltage to the desired value.

The circuit for the voltage regulator system is shown in Fig. 13. Alternator terminal voltage is applied to a potential transformer connected in V-V (TR1 and TR2). The reduced secondary voltage, proportional to the line-to-line voltage of the machine, is applied to a three phase dry-disc rectifier. The d-c output of the rectifier is supplied to the solenoid (regulator coil) in a series network with temperature compensating resistors and other auxiliary components including a stabilizing transformer TR3. The output of the rectifier is proportional to the average of the three line-to-line voltages. This arrangement makes use of three-phase voltage sensing. Three-phase sensing, though requiring a heavier and more complex regulator, gives better regulation with unbalanced loads.

Figure 14 is a vector diagram showing the pertinent current and voltage relationships in the circuit. The current transformer and mutual reactor of the equalizer are placed in the same phase. During parallel operation the primary winding of the reactor in the equalizer circuit carries current only if the division of current among alternators is improper. With only one alternator on the system the primary of the reactor is automatically short circuited. When the circuit breaker of an alternator is open the primary of the mutual reactor is short circuited. Thus the open circuit breakers on all the other alternators short circuit not only their own reactors but also that of the one alternator on the system. If an alternator delivers more than its share of reactive current, a 90 degree lagging current will flow through the primary of its mutual reactor and a corresponding 90 degree leading current will flow through the mutual reactors of the other alternators which causes the voltage regulators to take corrective action.
NOS. 1, 2 & 3 - ALTERNATOR LINE-TO-NEUTRAL VOLTAGES
NOS. (5-4), (4-6) & (6-5) - LINE-TO-LINE VOLTAGES
AT SECONDARY OF POTENTIAL TRANSFORMER
$|I_3|$ - CURRENT IN CT
$|I_3|X_m$ - VOLTAGE INDUCED IN SECONDARY OF MUTUAL REACTOR TR 4

FIG. 14 - GENERAL VECTOR DIAGRAM FOR VOLTAGE REGULATOR
Figures 15a, 15b, 15c, show vector diagrams for unity p.f., zero p.f. lagging, and zero p.f. leading current. For unity power-factor, the numerical average value of the voltage applied to the rectifier and consequently rectifier output is approximately the same for zero current (normal operations under balanced conditions) as for normal values of unity p.f. current.

For zero p.f. lagging, the numerical average value of the three voltages applied to the rectifier is increased which raises the voltage on the regulator coil and decreases excitation. For zero p.f. leading, the rectifier output decreases, increasing excitation. The effect of any intermediate power factor between zero and unity can be obtained by resolving the current vector into real and reactive components.

2. Speed Regulator

Figure 16 shows a schematic diagram of the constant speed drive. The output from the engine is geared to a rotating drum which houses two hydraulic elements comprised of an interconnected hydraulic pump and hydraulic motor. The output of the hydraulic motor is geared to the input shaft of the alternator through a free-wheeling clutch. The rated output speed is 6000 rpm. When the input speed is 6000 rpm, there is no pumping action. Deviations in the input speed from 6000 rpm cause the angle of the adjustable wobble plate to be such that the proper pumping action is obtained so that the output speed is 6000 rpm.

The position of the wobble plate is determined by a governor shown schematically in Fig. 16 which is actuated by an electrical solenoid comprised of three coils. These coils are the pilot valve operating coil WC, droop coil WD, and an overspeed coil OC, as shown in Fig. 17.

The pilot valve operating coil is energized through rectifiers
NOS. 4, 5, 6, & 7 CORRESPOND TO WIRE NUMBERS ON SCHEMATIC DIAGRAM FIG. 13
VOLTAGES (6-4), (5-6) & (4-5) ARE 3Φ OUTPUT VOLTAGES OF LINE TRANSFORMER BANK (TR1 & TR2, FIG. 13)
VOLTAGES (7-4), (5-7) & (4-5) ARE 3Φ VOLTAGES APPLIED TO A-C TERMINALS OF RECTIFIER

FIG. 15 - VECTOR DIAGRAMS FOR VOLTAGE REGULATOR - VARIOUS POWER FACTORS

WADC TR 52-49
DRIVE ALTERNATOR
INPUT
ALTERNATOR
INPUT
AIRCRAFT
CONSTANT
POWER OUTPUT
SPEED
ALTERNATOR
I
TACHOMETER
AND CONTROL
HYDRAULIC
CIRCUIT
TRANSMISSION
TO CONTROL
PISTON
PILOT VALVE
AND SOLENOID
ASSEMBLY
DROOP CIRCUIT

BLOCK DIAGRAM OF CONSTANT SPEED DRIVE AND ASSOCIATED EQUIPMENT

CONTROL PISTON MOVEMENT
VARIABLE DISPLACEMENT REVERSIBLE HYDRAULIC ELEMENT
ADJUSTABLE WOBBLE PLATE

DRIVE INPUT FROM ENGINE 2400 TO 9000 RPM

FIG. 16 - SCHEMATIC DIAGRAMS OF CONSTANT SPEED DRIVE

WADC TR 52-49
- 68 -
from a three-phase tachometer, and is so adjusted that when acting alone (coils WD and OC inactive), the output speed is maintained nominally at 6000 rpm.

The droop coils in a parallel system of alternators can be energized in various ways. In the early use of systems of parallel alternators, the droop coil WD was connected directly to the droop circuit of its respective alternator, by connecting output terminals 1 and 2 of the droop circuit to terminal D and C of the droop coil in the governor solenoid in Fig. 17, thus deliberately incorporating a droop characteristic such that the speed decreased with increasing real power load. This was done with a view toward effecting proper division of real power load between the alternators. This arrangement has two disadvantages: one is the decrease in frequency in going from no load to full load, the other is the extreme precision required to make the droop adjustments of all alternators identical. A much improved arrangement for dividing real power load without sacrificing accuracy in frequency has been introduced by the Air Materiel Command. This arrangement makes use of an equalizer circuit which is shown schematically in Fig. 18. With this arrangement there is no current in the droop coils except when there is unequal division of real power between alternators. The effect of such a current in the droop coils is to initiate corrective action which reduces this current to zero. Under equal real load conditions regardless of the magnitude of this load, only the pilot valve operating coil which is energized from the tachometer is effective in the operation of the governor. If the speed governor were ideal, departures from normal frequency would be transient existing only during periods in which the division of real power undergoes readjustment.
DROOP ADJUSTING RESISTOR

OUTPUT OF DROOP CIRCUIT

GOVERNOR DROOP SOLENOID

a - EQUIVALENT CIRCUIT

b - SIMPLIFIED EQUIVALENT CIRCUIT

FIG. 18 - SCHEMATIC DIAGRAM OF EQUALIZER CONNECTIONS FOR REAL LOAD DIVISION

WADC TR 52-49 - 65 -
The overspeed coil in the governor solenoid is energized by means of a centrifugal switch from an auxiliary d-c power system (usually 28 volts) when the output speed rises above 7500 rpm (125 percent rated speed). Under this condition the adjustable wobble plate in the constant speed drive is shifted to the full underdrive position and the plunger of the solenoid becomes latched in this position. In order to restore normal operation of the drive, it is necessary to reset the latch manually.

3. Equalizer Circuits

For efficient parallel operation of alternators it is necessary that both real and reactive load be distributed equally among the alternators. The present system employs both reactive and real power equalizers which are interconnections of the respective voltage and speed regulating systems between the paralleled alternators so that unbalances in load can be corrected automatically.

a. Reactive Power Equalizer

To achieve equal division of reactive load among alternators, the equalizer current transformers of the alternators are connected in series as shown in Fig. 19. When each alternator on the bus is delivering its share of current, the same current flows through all of the current transformers and zero current flows in the equalizer coils (mutual reactor primary, TRh, Fig. 13).

An analysis of the reactive load equalizer circuit is presented in the following.

1) Analysis of Reactive Load Equalizer Circuit

Figure 19 shows the schematic diagram for the interconnection of \( n \) current transformers each supplying the primary of a mutual reactor to
FIG. 19—SCHEMATIC DIAGRAM OF REACTIVE EQUALIZER CONNECTIONS

G IS THE CURRENT TRANSFORMER RATIO
form the reactive load equalizer circuit for an n-alternator system.

If the currents in each of the n machines are $I_1, I_2, \ldots, I_n$ and the current transformer ratios are all $c$, then $\frac{1}{c}I_1, \frac{1}{c}I_2, \ldots, \frac{1}{c}I_n$ are the currents in the n current transformers. If machine 1 has its load increased for any reason then $I_1$ will be different from the other (n-1) currents. Thus $I_1 \neq I_2 = I_3 = \ldots = I_n$.

If the currents are summed at points A and N in Fig. 19 two expressions are obtained.

$$\frac{1}{c}I_1 = i_1 + i$$

$$\frac{1}{c}I_n = i_n + i$$

Subtracting the second equation from the first gives

$$\frac{1}{c} (I_1 - I_n) = i_1 - i_n$$

The sum of the voltages around the loop containing the primaries of the n mutual reactors, if the impedance of the interconnecting leads is negligible is expressed by

$$i_1 Z_1 + i_2 Z_2 + \ldots + i_n Z_n = 0$$

or if $Z_1 = Z_2 = \ldots = Z_n$ this reduces to

$$i_1 + (n-1) i_n = 0$$

since $i_2 = i_3 = \ldots = i_n$

and on transposing $i_1 = -(n-1) i_n$

Substituting Eq. (8) in (5) yields

$$i_n = -\frac{1}{cn} (I_1 - I_n)$$

Substituting Eq. (9) in Eq. (8) we get

$$i_1 = \frac{(n-1)}{cn} (I_1 - I_n)$$
Equations (9) and (10) express the mutual reactor currents as functions of the difference current \((I_1 - I_n)\), which is in general a vector with both real and reactive components. Under normal conditions, only the reactive components of this difference affect the operation of the voltage regulator.

When only one alternator is on the system and its mutual reactor is not short circuited, the polarities of the current transformer and mutual reactor are correct if the voltage droops when reactive power is fed to the load. Only reactive component of current in the mutual reactor affects the voltage regulator. Hence, if \(I_m\) is the current in the mutual reactor

\[
|V| = |V_0| - |B| |I_m| \sin \theta
\]  

where:

- \(|V|\) = magnitude of terminal voltage,
- \(|V_0|\) = magnitude of reference voltage,
- \(\theta\) = angle by which the current lags the voltage,
- \(|I_m|\) = magnitude of mutual reactor current,
- \(|B|\) = constant which combines the ratio of the current transformer and the mutual reactance of the mutual reactor.

The quadrature component of each of the mutual reactor currents corresponds to \(|I_m| \sin \theta\) in equation (11) and can be determined from the quadrature component of the difference current \((I_1 - I_n)\) which is expressed by:

\[
|I_1| \sin \theta_1 - |I_n| \sin \theta_n
\]  

where:

- \(\theta_1\) = angle by which \(I_1\) lags the terminal voltage,
- \(\theta_n\) = angle by which \(I_n\) lags the terminal voltage.
Making use of Eqs. (9) and (10) and substituting Eq. (12) in Eq. (11), the terminal voltage of alternator \(1\) is expressed by

\[
|V| = |V_0| - K \frac{(n-1)}{n} (|I_1| \sin \Theta_1 - |I_n| \sin \Theta_n)
\]  

(13)

and that of the other \((n-1)\) alternators is expressed by

\[
|V| = |V_{on}| + K \frac{1}{n} (|I_1| \sin \Theta_1 - |I_n| \sin \Theta_n)
\]  

(14)

where:

- \(|V_0|\) = reference voltage for alternator \(1\),
- \(|V_{on}|\) = reference voltage for the \((n-1)\) alternators,
- \(K = \frac{|B|}{c}\), the amplification factor of the reactive load equalizing circuit.

2) Determination of Reactive Current Equalizer Amplification Factor

The value of the amplification factor \(K\) of regulator alternator D was determined by test. The equalizer connections from alternator D were disconnected so that all the secondary current from the current transformer flowed in the mutual reactor of that alternator. Alternator 1 was under-excited and used as a low power-factor load for alternator D and results of the test are shown in Table XVI. Figure 20 shows graphically the relationship between the terminal voltage, \(V\), and the quadrature component of current, \(|I| \sin \Theta\), based on the test results. This graph shows a straight line relationship. The magnitude of the amplification factor \(K\) is equal to that of the slope of this line. Although the slope of the line is negative, the factor \(K\) is positive and has a value of \(0.106\) per unit. The equation of the line in Fig. 20 is expressed by

\[
V = V_0 - K |I| \sin \Theta
\]  

(15)

According to the specification for this type of voltage regulator (MIL-R-5292A) the value of this constant is such as to give a drop of from
<table>
<thead>
<tr>
<th>Phase Voltage Volts</th>
<th>Phase Voltage P.U.</th>
<th>Current Amperes</th>
<th>Current P.U.</th>
<th>Power KW P.U.</th>
<th>Power KVARS P.U.</th>
<th></th>
<th>magnetic [I]</th>
<th>magnetic sin θ</th>
</tr>
</thead>
<tbody>
<tr>
<td>123.2</td>
<td>1.027</td>
<td>26.0</td>
<td>0.234</td>
<td>2.4</td>
<td>0.060</td>
<td>-9.57</td>
<td>-0.239</td>
<td>-0.228</td>
</tr>
<tr>
<td>120.3</td>
<td>1.003</td>
<td>2.0</td>
<td>0.018</td>
<td>1.92</td>
<td>0.048</td>
<td>-0.693</td>
<td>-0.017</td>
<td>-0.006</td>
</tr>
<tr>
<td>117.7</td>
<td>0.981</td>
<td>22.6</td>
<td>0.204</td>
<td>2.16</td>
<td>0.054</td>
<td>7.48</td>
<td>0.187</td>
<td>0.196</td>
</tr>
<tr>
<td>116.8</td>
<td>0.974</td>
<td>31.0</td>
<td>0.279</td>
<td>2.4</td>
<td>0.060</td>
<td>10.5</td>
<td>0.263</td>
<td>0.273</td>
</tr>
<tr>
<td>114.6</td>
<td>0.955</td>
<td>49.4</td>
<td>0.444</td>
<td>2.4</td>
<td>0.060</td>
<td>16.9</td>
<td>0.423</td>
<td>0.442</td>
</tr>
<tr>
<td>113.8</td>
<td>0.949</td>
<td>59.4</td>
<td>0.535</td>
<td>2.4</td>
<td>0.060</td>
<td>20.2</td>
<td>0.505</td>
<td>0.533</td>
</tr>
<tr>
<td>112.9</td>
<td>0.941</td>
<td>65.8</td>
<td>0.593</td>
<td>2.4</td>
<td>0.060</td>
<td>22.3</td>
<td>0.558</td>
<td>0.591</td>
</tr>
</tbody>
</table>
FIG. 28 - REACTIVE EQUALIZER DROOP CHARACTERISTIC

SLOPE = -1.16  K = 1.06

QUADRATURE CURRENT - (|I| \sin \theta) - PER UNIT

TERMINAL VOLTAGE - PER UNIT

WADC TR 52-49
5 to 7% rated voltage when the alternator is supplying half rated current at zero power-factor lagging. The value of 0.106 for K gives a drop of 5.3% rated voltage for the specified current condition.

3) **Errors in Reference Voltage and Reduction in System Capacity**

The total available capacity of a system in which paralleled alternators do not share load equally is less than the sum of the capacities of the individual machines, since one or more alternators will be overloaded before full system capacity is reached. Division of reactive load can be achieved by making use of voltage droop characteristics. This method is less desirable than one using an equalizer circuit, because the voltage droop method depends upon an appreciable variation of voltage with load.

In the equalizer method, a loss of available system capacity due to current unbalance can be caused by errors in the voltage regulator setting. These errors may be due to drifts such as are caused by changes in resistance of the regulator coil circuit or may be due to inaccuracies in the original settings. Specifications used by the Air Force (MIL-R-5292A) place a limit of 2.5 percent in the deviation from rated value due to inaccuracies in the regulator itself.

Suppose that in a system of \( n \) alternators operating in parallel, the regulator of one alternator is in error by 2.5 percent and the regulators on the other alternators are at their proper setting. For this condition the capacity of the system can be determined by calculating the total system load at the time that any alternator reaches its rated capacity.

The expression relating bus voltage, regulator setting and equalizer currents has been derived for each of the \( n \) alternators and are as follows:

\[
|V| = |V_{01}| - K \left( \frac{(n-1)}{n} \right) (|I_1| \sin \theta_1 - |I_n| \sin \theta_n)
\]  

(13)
and for each of the other \((n-1)\) alternators
\[
|V| = |V_{on}| + K \frac{1}{n} (|I_1| \sin \theta_1 - |I_n| \sin \theta_n)
\] (14)

Since the terminal voltages of the two machines must be equal and if Eq. (14) is subtracted from Eq. (13), there results
\[
0 = |V_{ol}| - |V_{on}| - K (|I_1| \sin \theta_1 - |I_n| \sin \theta_n)
\]
or
\[
|I_1| \sin \theta_1 - |I_n| \sin \theta_n = \frac{1}{K} (|V_{ol}| - |V_{on}|)
\] (16)

The KVA capacity will be computed for (1) the regulator setting in alternator 1 2.5% below normal and (2) for a setting 2.5% above normal with the regulators of each of the other alternators normal. It is also assumed that the alternators share real load equally and that none of the alternators be overloaded.

An example for the case where \(|V_{ol}|\) is set 2.5% low is as follows:
\[
|I_1| \sin \theta_1 - |I_n| \sin \theta_n = \frac{1}{0.106} (-0.025)
\]
\[
|I_1| \sin \theta_1 - |I_n| \sin \theta_n = -0.236
\]
when \(|V_{ol}|\) is low \(|I_n| \sin \theta_n = 0.661\) (current is 1.00 p.u. at .75 P.F.)
and
\[
|I_1| \sin \theta_1 = 0.425
\]
The total quadrature current supplied to the load
\[
|I_L| \sin \theta_L = |I_1| \sin \theta_1 + (n-1) |I_n| \sin \theta_n
\]
If \(n = 4\)
\[
|I_L| \sin \theta_L = 0.425 + 1.98 = 2.41
\]
The in-phase component of load current is
\[
|I_L| \cos \theta_L = 4 \times .75 = 3.00
\]
Neglecting the change in system voltage which is 0.6 percent, we get for the
total load power in per unit

\[ S_L = 3.00 + j 2.41 \]

\[ = 3.85/38.8^\circ \]

and the percent installed capacity is

\[ \frac{3.85}{3.00} \times 100 = 96.2\% \]

In Table XVII are tabulated the values of system capacity for values of \( n \) from 2 to 6 and for settings of \( V_{ol} \) 2.5 percent above and 2.5 percent below normal values.

<table>
<thead>
<tr>
<th>Number of Alternators, ( n )</th>
<th>( V_{ol} = 1.025 \text{ P.U.} )</th>
<th>( V_{ol} = 0.975 \text{ P.U.} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>% Cap.</td>
<td>( V )</td>
<td>% Cap.</td>
</tr>
<tr>
<td>2</td>
<td>93</td>
<td>1.013</td>
</tr>
<tr>
<td>3</td>
<td>91</td>
<td>1.008</td>
</tr>
<tr>
<td>4</td>
<td>89</td>
<td>1.006</td>
</tr>
<tr>
<td>5</td>
<td>88</td>
<td>1.005</td>
</tr>
<tr>
<td>6</td>
<td>88</td>
<td>1.004</td>
</tr>
<tr>
<td>( \infty )</td>
<td>86</td>
<td>1.000</td>
</tr>
</tbody>
</table>

If the regulator setting is 2.5 percent low, the reduction in the system capacity is small particularly as the number of generating units is increased. However, if the regulator setting is 2.5 percent high, the reduction in system capacity becomes sizeable, approaching 86 percent as a limit. An increase in the value of \( K \) decreases the reduction in system capacity resulting from errors in reference voltage. However, there is a hazard of increasing \( K \) to a value where hunting of the voltage regulating system would
result. The determination of the optimum value of the amplification factor $K$ is complex, but is a problem which deserves further study.

b. **Real Power Equalizer**

The equalizer circuit for dividing real power among alternators is shown in Fig. 18a. For an $n$ alternator system there are $n$ branches in parallel and each branch is comprised of the droop circuit in series with its own solenoid droop coil. Thus when all of the alternators are carrying the same real load, each droop circuit has the same output voltage and no current flows in any of the branches. If one alternator should pick up an increment of real load and each of the other machines still carry the same loads, current will flow in each branch in such a direction as to correct the unbalance and tend to restore the equal load division.

1) **Analysis of Real Load Equalizer Circuit**

Let $I_{pl}$ be the in-phase component of current being supplied by alternator 1 and $I_{pn}$ be the in-phase component of current being supplied by each of the other alternators. $E_{o1}$ is the D.C. voltage across the resistor $R_2$ in Fig. 17 in the droop circuit of alternator 1, and $E_{on}$ is the corresponding voltage for each of the $(n-1)$ machines. It is shown in Appendix III-B that $E_{o1}$ is substantially proportional to $I_{pl}$ as $I_{ql}$ the reactive current has negligible effect on the division of real power. If the constant of proportionality is called $M$, then

$$E_{o1} = M I_{pl} \quad (17)$$

$$E_{on} = M I_{pn} \quad (18)$$

Figure 18a shows the circuit to be analysed. This circuit can be reduced to the simple network of Fig. 18b.

If $R$ is the resistance of the droop adjusting resistor of each
alternator (R₁₄ in Fig. 17), and R₄ is the resistance of each solenoid droop coil, then the simplified circuit yields:

\[ i₁ = \frac{(n-1)(E_{o1} - E_{on})}{n(R + R₄)} \]

where \( i₁ \) is the current in the droop coil of machine 1.

From Equations (17) and (18)

\[ E_{o1} - E_{on} = M(I_{p1} - I_{pn}) \]

so

\[ i₁ = \frac{(n-1)M}{n(R + R₄)} (I_{p1} - I_{pn}) \]

and the voltage across the droop coil of alternator 1 is expressed by:

\[ V_{d1} = -\frac{(n-1)R₄}{n(R + R₄)M} (I_{p1} - I_{pn}) \]  

The frequency of an alternator is almost a linear function of the voltage across the droop coil and if \( f₀ \) is the reference frequency (i.e. the frequency when the droop coil voltage, \( V_d \), is zero) then the frequency can be expressed by

\[ f = f₀ + N V_d \]

where \( N \) is a constant.

The system frequency of the \( n \)-machine system in terms of the reference frequency of machine 1 and its droop coil voltage:

\[ f = f_{o1} + N V_{d1} \]

If Eq. (22) is substituted into Eq. (23)

\[ f = f_{o1} - N M \frac{R₄}{(R + R₄)} \frac{(n-1)}{n} (I_{p1} - I_{pn}) \]  

If a new constant \( K' \) is defined by

\[ K' = N M \frac{R₄}{(R + R₄)} \]
then Eq. (24) is reduced to

\[ f = f_0 - K' \frac{(n-1)}{n} (I_{pl} - I_{pn}) \]  \hspace{1cm} (25)

Eq. (25) states that the system frequency will be constant at the value of the reference frequency \( f_0 \), if the difference current, \( I_{pl} - I_{pn} \) is zero at any system load. Zero difference current can be achieved only if all speed governors are set for the same reference frequency, \( f_0 \), by the proper adjustment of the engineer's rheostat \( R_3, R_5 \) Fig. 17, on each unit. When there is an inaccuracy in the setting of one of these rheostats, a sustained difference current will exist and a loss of system capacity will result.

2) **Determination of In-Phase Current Equalizer Amplification Factor**

Tests were made on alternator D and its associated constant speed drive to determine the speed droop characteristics for the following conditions:

1. Droop circuit disconnected
2. Droop circuit adjusted for minimum droop
3. Droop circuit adjusted for maximum droop.

1. For the first test, the droop circuit was opened in order to render it ineffective and to determine the droop inherent in the pilot valve control arrangement. With alternator D carrying no load, the reference frequency was adjusted to 400 cycles per second. Alternator D was then loaded with a unity power-factor load and measurements of frequency and power were made. The results are shown in Table XVIII and are plotted in Fig. 21.

2. The droop coil of alternator D was connected to its own droop circuit and the resistance \( R_4 \) shown in Fig. 17 was adjusted to its maximum value so that the droop would be a minimum as the load on alternator D was...
<table>
<thead>
<tr>
<th>Condition of Test</th>
<th>Frequency Cycles</th>
<th>Per Unit</th>
<th>Three-Phase Power KW</th>
<th>Per Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unity Power-Factor</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Droop Circuit</td>
<td>399</td>
<td>.998</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Open</td>
<td>398</td>
<td>.995</td>
<td>9.95</td>
<td>.219</td>
</tr>
<tr>
<td></td>
<td>395</td>
<td>.988</td>
<td>22.1</td>
<td>.553</td>
</tr>
<tr>
<td></td>
<td>393</td>
<td>.983</td>
<td>28.8</td>
<td>.72</td>
</tr>
<tr>
<td></td>
<td>388.5</td>
<td>.971</td>
<td>36.8</td>
<td>.92</td>
</tr>
<tr>
<td></td>
<td>399.7</td>
<td>.999</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>397.5</td>
<td>.994</td>
<td>3.84</td>
<td>.096</td>
</tr>
<tr>
<td>Min. Droop Adjustment</td>
<td>393.8</td>
<td>.985</td>
<td>11.6</td>
<td>.29</td>
</tr>
<tr>
<td></td>
<td>388</td>
<td>.970</td>
<td>20.2</td>
<td>.505</td>
</tr>
<tr>
<td></td>
<td>380.5</td>
<td>.951</td>
<td>29.5</td>
<td>.738</td>
</tr>
<tr>
<td></td>
<td>371</td>
<td>.928</td>
<td>38.2</td>
<td>.945</td>
</tr>
<tr>
<td></td>
<td>398.7</td>
<td>.998</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>384.5</td>
<td>.961</td>
<td>10.1</td>
<td>.252</td>
</tr>
<tr>
<td>Max. Droop Adjustment</td>
<td>371</td>
<td>.928</td>
<td>19.7</td>
<td>.492</td>
</tr>
<tr>
<td></td>
<td>355</td>
<td>.889</td>
<td>29.3</td>
<td>.732</td>
</tr>
<tr>
<td></td>
<td>335</td>
<td>.838</td>
<td>37.7</td>
<td>.942</td>
</tr>
</tbody>
</table>
FIG. 21 - FREQUENCY DROOP CHARACTERISTICS
increased. Measurements of real power versus frequency were repeated.

3. These measurements were again repeated with the resistance $R_4$ adjusted for its minimum value in order to obtain maximum droop.

In Fig. 21, the curves are approximated by straight lines in order to obtain constant values for the amplification factors $K'$. On that basis $K' = 0.063$ per unit when the circuit is adjusted for minimum droop; and $K' = 0.158$ per unit when the circuit is adjusted for maximum droop.

3) **Output Voltage of Frequency Droop Circuit**

Tests were made on alternator D to determine the relationship between the d-c output voltage of the frequency droop circuit and the real power output at unity power-factor. The data obtained in these tests are listed in Table XIX, and are shown by the circled points in Fig. 22.

<table>
<thead>
<tr>
<th>Unity Power Factor</th>
</tr>
</thead>
</table>

### Table XIX

**TESTS OF FREQUENCY DROOP RESPONSE**

<table>
<thead>
<tr>
<th>Three-Phase Power</th>
<th>Output Voltage, $E_0$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>KW</strong></td>
<td><strong>Per Unit</strong></td>
</tr>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>8.65</td>
<td>0.216</td>
</tr>
<tr>
<td>15.1</td>
<td>0.377</td>
</tr>
<tr>
<td>26.2</td>
<td>0.655</td>
</tr>
<tr>
<td>34.3</td>
<td>0.856</td>
</tr>
<tr>
<td>40.0</td>
<td>1.000</td>
</tr>
</tbody>
</table>

4) **Effect of Error in Reference Frequency**

Drifts in the reference frequency can result from changes in the resistance of the tachometer and the pilot valve control winding circuit. Perhaps the most common cause for variations in resistance is a change in the temperature of the pilot valve control winding. The resistance of this
CURVES SHOWING DROOP
CIRCUIT OUTPUT VOLTAGE
AS A FUNCTION OF IN-PHASE
CURRENT FOR VARIOUS VALUES
OF QUADRATURE CURRENT

OBTAINED FROM TEST
I_q = 0

FIG. 22 - DROOP CIRCUIT RESPONSE TO IN-PHASE CURRENT

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winding increases with increasing temperature and if the reference frequency is adjusted with the coil at one temperature, an increase in temperature will produce an increase in the reference frequency. In the studies at Wright Field drifts of more than 5 percent in the reference frequency were observed during the warm-up period (approximately one-half hour).

Thus with a system of any number of alternators, if all the alternators are synchronized to the bus cold and at about the same time, with the frequency of each adjusted to 400 cycles and are left unattended, the system frequency will rise to a value exceeding 420 cycles because of heating. For some types of load this value of frequency would be excessive.

Consider an n-alternator system in which (n-1) alternators are operating in parallel at a steady temperature and at the proper frequency. Suppose that the remaining alternator (alternator 1) is then synchronized to the system without having warmed up. By the time this alternator reaches a steady temperature, its reference frequency will have increased by more than 5 percent. This alternator will assume more than its share of the real power load and by the time it is fully loaded, the remaining alternators will be carrying less than their rated load. Any increase in the system load will cause alternator 1 to become overloaded. Thus the capacity of the system is reached when alternator 1 attains rated load. In the following, the capacity of the system is determined when the reference frequency of one alternator deviates from its proper value.

The expressions for the system frequency, in terms of the individual machine no load settings, and the difference current are:

For alternator 1

\[ f = f_{01} - K' \frac{1}{n} (I_{pl} - I_{pn}) \]  

(26)

For each of the other (n-1) alternators

\[ f = f_{on} + K' \frac{1}{n} (I_{pl} - I_{pn}) \]  

(27)
The system frequency must satisfy both equations and if $f$ is eliminated

$$I_{pl} - I_{pn} = \frac{1}{K} (f_{o1} - f_{on}) \tag{28}$$

If $f_{o1}$ is high, machine 1 will reach its rating first and the other machines will carry less than their ratings; while if $f_{o1}$ is low, then the $(n-1)$ machines will be at rated load first and only machine 1 will carry less than its rating. It is obvious that there is less capacity reduction in the latter case than in the former.

Suppose that the reference frequency, $f_{o1}$, of alternator 1 has drifted to 1.05 per unit (420 cycles) and that the droop circuits are adjusted for maximum droop i.e. $K' = 0.158$ per unit. Under this condition when alternator 1 carries rated load at 0.75 power factor we have

$$I_1 = 0.75 - j 0.66 \text{ per unit}, \text{ where}$$

$$I_{pl} = 0.75$$

From Eq. (27) the in-phase current in the other alternators is expressed by

$$I_{pn} = I_{pl} - \frac{1}{K} (f_{o1} - f_{on})$$

$$= 0.75 - \frac{0.05}{0.158} = 0.434 \text{ per unit}$$

If the reactive current equalizers are functioning properly the total current in each of the other alternators is expressed by

$$I_n = 0.434 - j 0.66 \text{ per unit}.$$ 

The total generated power then becomes

$$S_{TH} = 0.75 + (n-1) 0.434 + j 0.66 \text{ } n$$

If the reference frequency $f_{o1}$ of alternator 1 were reduced to 0.95 per unit it can be shown that the total generated power becomes

$$S_{TL} = 0.434 + (n-1) 0.75 + j 0.66 \text{ } n$$
The reduced values of system capacity due to errors in the reference frequency of one alternator are listed in Table XX.

Table XX

EFFECT OF FREQUENCY ERROR ON SYSTEM CAPACITY

<table>
<thead>
<tr>
<th>Number of Alternators, n</th>
<th>( f_{o1} = 1.05 ) per unit</th>
<th>( f_{o1} = 0.95 ) per unit</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>% Capacity</td>
<td>( f )</td>
</tr>
<tr>
<td>2</td>
<td>0.89</td>
<td>1.025</td>
</tr>
<tr>
<td>3</td>
<td>0.85</td>
<td>1.017</td>
</tr>
<tr>
<td>4</td>
<td>0.84</td>
<td>1.013</td>
</tr>
<tr>
<td>5</td>
<td>0.83</td>
<td>1.010</td>
</tr>
<tr>
<td>6</td>
<td>0.82</td>
<td>1.008</td>
</tr>
<tr>
<td>( \infty )</td>
<td>0.79</td>
<td>1.000</td>
</tr>
</tbody>
</table>

The reduction in the system capacity can be decreased by increasing the amplification factor \( K' \) at the possible risk of incurring instability in the speed governing system. In the analysis upon which the values in Table XX are based, a deviation in the reference frequency of 5 percent was taken. An excess of 5 percent in the reference frequency can easily result from increased resistance in the pilot value control coil due to increased temperature. Rather than increasing the value of \( K' \) to minimize the reduction in system capacity, the effect of temperature on the resistance of the control circuit should be reduced. This might be accomplished in the existing circuit by means of compensating resistors with negative temperature coefficient or by devising some other method of regulation in which temperature effects are negligible.

4. **Malfunctioning of Regulating Systems**

The characteristics of the systems which regulate the voltage, frequency, real power division, and reactive power division have been described in the foregoing sections with emphasis on normal operation. The
results of combat damage or other accidents can cause the regulating systems to malfunction. The voltage regulating system which also equalizes the reactive current would malfunction under the following conditions:

a. Abnormal excitation of one of several alternators operating in parallel
   1) Ceiling excitation
   2) Loss of field excitation
b. Opening of reactive current equalizer connection
c. Opening of current transformer in reactive load equalizing circuit

The frequency regulating system would malfunction under the following conditions:

d. Failure of tachometer or tachometer circuit
e. Short circuit of frequency droop circuit
f. Failure of potential transformers in frequency droop circuit.

a. Abnormal Excitation

There are two extremes of abnormal alternator field excitation. One of these extremes occurs when the field excitation goes to ceiling value and if sustained at that value, the exciters and alternators would overheat. To prevent such damage, the exciter protection relay is set to operate in about 2 seconds causing the armature circuit breaker to open and the exciter field circuit to be de-energized. The other extreme is that of complete loss of field excitation. Under this condition the alternator draws excessive reactive power from the other alternators on the system and its real power output becomes limited to 25% of its rated value or less.

1) Ceiling Excitation

If the voltage to the voltage regulator coil is reduced to a low
value, the regulator will raise the field excitation of the alternator to ceiling value. Such a reduction in the voltage to the regulator coil may be caused erroneously by: (1) loss of one or more leads connecting to the V-V transformer (2) an open circuit or (3) a substantial increase in resistance in the regulator coil circuit (4) a complete or partial short circuit in the regulator coil (5) damage to the regulator mechanism. Ceiling excitation may also result from a short circuit of the regulating resistor in series with the exciter field.

When the excitation of one alternator operating in parallel with others goes to its ceiling value, that alternator delivers more than its share of reactive power. The reactive power equalizer circuit indicates a deficiency of reactive power output from the other alternators, causing the exciters of these machines to increase excitation. This action sustains the bus voltage at a value above normal. Figure 25 shows the calculated effect of ceiling excitation on bus voltage graphically. This figure shows that, in the case of 2 alternators in parallel, with one at ceiling excitation, the bus voltage can have a value which is 13 percent above normal. The time required (2 seconds) for the exciter protection relay to clear the defective alternator, thus restoring the system voltage to normal, is rather long and some faster means of protection for this type of failure is desirable.

Suppose that one alternator of an n-alternator system has its excitation at ceiling value. There are three characteristics which must be considered in calculating the system voltage and division of reactive power between alternators. They are as follows:

a) Volt-ampere characteristic of the faulty alternator
b) Voltage regulator characteristic
c) Load characteristic.

a) Volt-Ampere Characteristic of Faulty Alternator

In the faulty alternator the voltage regulator is not effective and the excitation goes to ceiling value. The relationship between the alternator terminal voltage and its armature current is determined by the ceiling voltage and the saturated synchronous reactance of the alternator. This relationship was obtained by using the results of alternator tests. Figure 23 shows the air-gap line, no load saturation curve, and zero power-factor curves calculated for different load currents with the aid of the Potier triangle. The characteristics shown in Fig. 23 were obtained by methods outlined previously. Once the no load saturation curve and the Potier triangle are obtained, the zero power-factor curves may be constructed because all Potier triangles for a given machine are practically similar.

The volt-ampere characteristic in Fig. 24 was determined from the characteristics shown in Fig. 23 using values which correspond to ceiling field current. The slope of the line gives a magnitude of saturated direct-axis synchronous reactance. This value corresponds to zero power-factor conditions, but is used throughout this analysis as the over-excited alternator operates at a lagging power factor and since the field current is a maximum, the degree of saturation changes only slightly at different values of power factor.

The voltage equation for an alternator (neglecting saliency and armature resistance) is written as follows:

\[ V = E_1 - j X_{dl} I_1 \]  

(29)

The subscript 1 refers to alternator 1 which has the faulty regulator. In that equation:
FIG. 23 - ZERO P.F. CHARACTERISTICS

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FIG. 24 - SATURATED VOLT-AMPERE CHARACTERISTIC

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- 90 -
V = terminal voltage

$E_1$ = internal voltage

$X_{dl}$ = direct-axis reactance

$I_1$ = armature current of alternator 1

b) **Voltage Regulator Characteristic**

Since the excitation of alternator 1 in the n-alternator system is forced to ceiling, its reactive component of current is increased. The expression for the current in the mutual reactor of any one of the (n-1) alternators is expressed by

$$i_n = - \frac{1}{c} \frac{(I_1 - I_n)}{n} \quad (9)$$

In this case, the difference $(I_1 - I_n)$ is such that the (n-l) alternators in order to correct the unbalance increase their excitation thus raising the terminal voltage. The bus voltage must satisfy equation (14).

$$|V| = |V_{on}| + \frac{K}{n} (|I_1| \sin \theta_1 - |I_n| \sin \theta_n) \quad (14)$$

which can be rewritten as follows because the in-phase components are equal.

$$|V| = |V_{on}| + j \frac{K}{n} (I_1 - I_n) \quad (30)$$

c) **Load Characteristic**

The current drawn by the load is the sum of the currents supplied by the n machines so

$$I_L = I_1 + (n-1) I_n \quad (31)$$

since $I_2 = I_3 = \ldots = I_n$

If $Z_L$ is the load impedance, then the bus voltage, is expressed by

$$V = Z_L [I_1 + (n-1) I_n] \quad (32)$$

The load impedance expressed in complex form is $Z_L (\cos \theta_L + j \sin \theta_L)$

where $\theta_L$ is positive when $I_L$ lags $V$. 

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d) Overvoltage and Power Equations

Eqs. (29), (30), and (32) yield the following expression for the bus voltage.

$$V = \frac{\frac{|E_1|}{X_{dl}} \cos \delta_1 + \frac{|V_0|}{K} (n-1) + j \frac{|E_1|}{X_{dl}} \sin \delta_1}{\frac{1}{X_{dl}} + \frac{n-1}{K} + \frac{\sin \Theta_1}{n |Z_L|} + j \frac{\cos \Theta_1}{n |Z_L|}} = |V| \ (1 + j0) \ (33)$$

The angle $\delta_1$ is the torque angle and is the angle by which the internal voltage of the alternator 1 leads the terminal voltage. The vertical bars signify magnitude.

Both the numerator and denominator of equation (33) are complex numbers. Since the bus voltage was assumed to lie along the real axis, the quotient of the two complex numbers must be real, which means $V$ is given by either the quotient of the reals or imaginaries. For the alternator which has gone to ceiling excitation the angle $\delta_1$ is small (about 7°), which means $\cos \delta_1$ may be assumed equal to unity with negligible error. Therefore, the quotient of reals can be approximated as

$$V = \frac{|E_1|}{X_{dl}} + \frac{|V_0|}{K} (n-1) + j \frac{|E_1|}{X_{dl}} \sin \delta_1$$

Equations for real and reactive power may be derived. The real power supplied by each of the $n$ machines is given by

$$P_1 = P_n = \frac{|V|^2}{n} \frac{\cos \Theta_1}{|Z_L|} \ (35)$$

The expressions for the reactive power from the machine which has gone to ceiling excitation and from the other machines on the system are

$$Q_1 = |V| \left[ \frac{|V|}{n} \frac{\sin \Theta_1}{|Z_L|} + \frac{(n-1) (|V| - |V_0|)}{K} \right] \ (36)$$
and

\[ q_n = |V| \left[ \frac{|V| \sin \theta_n}{n |Z_L|} - \frac{|V| - |V_0|}{K} \right] \]  \hspace{1cm} (37)

The formulas for the real and reactive components of the currents are obtained by dividing the expressions for \( P \) and \( Q \) by \( V \) and changing the sign of the reactive components.

e) Calculations and Results

Numerical values for the constants which appear in the preceding equations were determined by test. Base voltage and base current are 120 volts and 111.1 amperes respectively.

**Constants**

- \( E_1 = \) ceiling excitation = 158 volts = 1.32 per unit
- \( V_0 = \) voltage regulator setting = 120 volts = 1.0 per unit
- \( K = \) regulator coefficient = 0.106 per unit
- \( X_{dl} = \) saturated direct-axis synchronous reactance = 0.15 per unit

The results of the calculations for a 2-alternator system are shown in Table XXI. Figure 25 is a graph of terminal voltage versus reactive power delivered by a normal alternator. It may be seen that under most load conditions, the normal alternators receive reactive power. The point corresponding to full load zero power-factor (each alternator carrying rated current before regulator failure), has been included to show the end point. The straight line representing various load and power factor conditions for a 4-alternator system has also been included in Fig. 25.

Table XXI shows that the reactive power from the alternator with the faulty regulator lies in a range between 19 and 23 kilovars per phase for the two-alternator system. Oscillographic measurements showed that the
<table>
<thead>
<tr>
<th>Original Load on Each Alternator</th>
<th>V Voltage</th>
<th>P₁ = P₂</th>
<th>Q₁</th>
<th>Q₂</th>
<th>I₁</th>
<th>I₂</th>
</tr>
</thead>
<tbody>
<tr>
<td>P.U.</td>
<td>P.U. Volts</td>
<td>P.U. KW/Ø</td>
<td>P.U. KVARS/Ø</td>
<td>P.U. AMPS</td>
<td>P.U. AMPS</td>
<td></td>
</tr>
<tr>
<td>1.00</td>
<td>1.086 130.2</td>
<td>0.885 11.8 1.667</td>
<td>22.2</td>
<td>-0.108</td>
<td>-1.432</td>
<td>1.735</td>
</tr>
<tr>
<td>0.75</td>
<td>1.097 131.6</td>
<td>0.677 9.01 1.605</td>
<td>21.4</td>
<td>-0.412</td>
<td>1.548</td>
<td>1.585</td>
</tr>
<tr>
<td>0.50</td>
<td>1.109 133.0</td>
<td>0.461 6.14 1.553</td>
<td>20.7</td>
<td>-0.714</td>
<td>9.88</td>
<td>1.464</td>
</tr>
<tr>
<td>0.25</td>
<td>1.118 134.1</td>
<td>0.234 3.11 1.460</td>
<td>19.5</td>
<td>-1.043</td>
<td>13.9</td>
<td>1.325</td>
</tr>
<tr>
<td>0.00</td>
<td>1.132 136.0</td>
<td>0</td>
<td>1.448 18.9</td>
<td>1.418</td>
<td>18.9</td>
<td>1.25</td>
</tr>
</tbody>
</table>

LOAD POWER FACTOR = .75

LOAD POWER FACTOR = 1.00

| 1.00                             | 1.132 136.0 | 1.281 17.08 1.418 | 18.9 | 1.418 | 18.9 | 1.690 | 187 | 1.690 | 187 |
| 0.75                             | 1.132 136.0 | 0.960 12.8 1.418 | 18.9 | 1.418 | 18.9 | 1.512 | 168 | 1.512 | 168 |
| 0.50                             | 1.132 136.0 | 0.641 8.54 1.418 | 18.9 | 1.418 | 18.9 | 1.370 | 152 | 1.370 | 152 |
| 0.25                             | 1.132 136.0 | 0.320 4.26 1.418 | 18.9 | 1.418 | 18.9 | 1.284 | 142 | 1.284 | 142 |
| 0.00                             | 1.132 136.0 | 0 | 1.418 | 18.9 | 1.418 | 18.9 | 1.25 | 139 | 1.25 | 139 |
FIG. 25 - LOAD CHARACTERISTIC - CEILING EXCITATION

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faulty alternator delivered approximately 18 kilovars per phase. Even without any real load, the armature current exceeds rated value. Also for loads at or near unity power-factor, the other alternator on the system has excessive armature current because not only is it supplying a real power component of current but it is also receiving a reactive component.

The bus voltage of a 4-alternator system does not rise to as high a value as that for a 2-alternator system.

2) Loss of Field Excitation

An alternator may lose its field excitation due to an open circuit in either its own field circuit or in the exciter field circuit. A salient-pole alternator without field excitation can carry some real load without losing synchronism when operating in parallel with other synchronous machines. The expression for the real power delivered by a salient-pole alternator is given below:

\[
P = \frac{|V|}{|E|} \sin \delta + \frac{X_d - X_q}{2X_dI_q} |V|^2 \sin 2\delta \quad (38)
\]

where:

\( V \) = the terminal voltage of the alternator

\( E \) = the internal voltage

\( X_d, X_q \) are, respectively, the direct-axis and quadrature-axis unsaturated synchronous reactances

\( \delta \) = the angle between \( V \) and \( E \)

When the alternator loses its excitation, \( E \) becomes zero, and the expression for real power becomes

\[
P = \frac{X_d - X_q}{2X_dI_q} |V|^2 \sin 2\delta \quad (39)
\]
The maximum electrical load which this alternator can supply to the system without falling out of synchronism is

\[ P_{\text{max}} = \frac{X_d - X_q}{2X_dX_q} |V|^2 \]  

(40)

If the prime mover input to the unexcited alternator exceeds the value of \( P_{\text{max}} \) in Eq. (40) (plus the rotational losses), the alternator will be driven above synchronous speed. In this sense synchronous speed is considered proportional to the system frequency whether the frequency is normal or abnormal. If there were no induction generator effects the equation for the real power under this condition would be expressed by

\[ P = \frac{X_d - X_q}{2X_dX_q} |V|^2 \sin 2 \left[ 2\pi(f_1 - f) \right] t \]  

(41)

where:

- \( f \) = system frequency (corresponds to synchronous speed)
- \( f_1 \) = frequency corresponding to speed of faulty alternator

However, there are currents induced in the damper windings embedded in the field-pole faces. The frequency of these currents is the difference frequency \((f_1 - f)\) and their effect increases the electrical output of the alternator by virtue of induction generator effect.

The effect of opening the field circuit of one alternator when operating in parallel with another is shown in the oscillograms of Fig. 27 and Fig. 28. In the tests from which these oscillograms were obtained the field of alternator D was opened as indicated in Fig. 26. Alternator A was operated in parallel with alternator D in these tests with the real load division and reactive load division equalizer circuits in operation.

The load conditions for Fig. 27 and Fig. 28 on alternator D just prior to opening its field are shown in Table XXII.
FIG. 26 - CIRCUIT FOR LOSS OF FIELD TESTS.
Fig. 27 Effect of Opening Exciter Field of One of Two Alternators In Parallel. System Load 24.4 KVA, 0.65 p.f.
In Fig. 27 the traces of reactive power, bus voltage, and armature current are quite steady after the field of alternator D is opened. This shows that synchronism is maintained. The same traces in Fig. 28 show slight oscillations which indicate that synchronism is barely lost. Alternator D is driven at about 2 revolutions per second faster than alternator A. A more pronounced case in which synchronism is lost after the opening the field is shown by the oscillograms in Fig. 29. Alternator D carried a load of 38 KVA at a power factor of 0.71. The traces in Fig. 29 show marked oscillations and that alternator D is driven at about 6 revolutions per second faster than alternator A.

The condition under which the oscillograms in Fig. 27 were obtained might be considered the limiting condition for which synchronism is maintained when the field of alternator D is opened. Opening the field of an alternator produces a reduction in the system voltage due to the effect of the reactive load equalizer circuit. Fig. 27 shows that the voltage is 0.9 per unit after the field is opened, and Fig. 28 shows the voltage to vary between 0.6 per unit and 0.9 per unit. As the load used in these tests...
was passive its magnitude varies approximately as the voltage squared and a reduction in the real and reactive loads occurred after the field was opened. Hence, for Fig. 27 the real power after the field is opened is

\[ P = 0.20 \times (0.9)^2 = 0.16 \text{ p.u.} \]  

(42)

In order to approximate the load for the conditions of Fig. 28 the average of 0.6 and 0.9 is taken and is 0.75 p.u. Hence the real power for this condition is

\[ P = 0.30 \times (0.75)^2 = 0.17 \text{ p.u.} \]  

(43)

These values of real power indicate that the value of real load beyond which synchronism is lost is 0.16 per unit for the two alternators under test. This can also be verified by using Eq. (40). The values to be used in Eq. (40) are:

\[ X_d = 1.50 \text{ p.u.} \]

\[ X_q = 0.91 \text{ p.u.} \]

\[ |V| = 0.9 \text{ p.u. for Fig. 27} \]

Hence for the voltage recorded in Fig. 27

\[ P_{\text{max}} = \frac{1.50 - 0.91}{2 \times 1.50 \times 0.91} (0.9)^2 = 0.175 \text{ p.u.} \]  

(44)

Eq. (44) is in good agreement with Eq. (42) and Eq. (43).

The real power limit for sustained synchronism on opening the field of one alternator in the 2-alternator system investigated at Wright Field is approximately 0.16 per unit or about 6.4 kW. However for a system with more than two alternators the reduction in the system voltage is less and as a result a larger real power load can be delivered by the alternator with the open field without loss of synchronism.

a) Analysis of Performance with Open Alternator Field

In the case of an alternator losing its field excitation, the
effect of the equalizer circuit is opposite to its effect in the case of ceiling excitation. In order to analyze the effect of an open alternator field under load conditions during which synchronism is maintained, Eq. (34) which was derived for the case of ceiling excitation can be modified by letting $E_1 = 0$ and by substituting $(X_{du} + X_q)/2$ for $X_{dl}$ where:

$$X_{du} = \text{unsaturated direct-axis synchronous reactance}$$

$$X_q = \text{unsaturated quadrature-axis synchronous reactance}.$$  

The substitution of $(X_{du} + X_q)/2$ for $X_{dl}$ is based on the fact that under the conditions for loss of field the amount of magnetic saturation is small and also that the torque angle $\delta$, approaches 45 degrees as the real power load approaches 0.17 per unit beyond which loss of synchronism results. Eq. (34) then is reduced to

$$|V_o|\left(\frac{n-1}{2K}\right) = \frac{2}{X_{du} + X_q} + \frac{n-1}{K} + \frac{\sin \theta_L}{|Z_L|}.$$  

(45)

Rearranging the data in Table XXII for Fig. 27 we have

$$\sin \theta_L = 0.76$$

$$|Z_L| = \frac{40}{24.1} = 1.64 \text{ p.u.}$$

The other quantities are as follows:

$$|V_o| = 1.00 \text{ p.u.}$$

$$X_{du} = 1.50 \text{ p.u.}$$

$$n = 2$$

$$X_q = 0.91 \text{ p.u.}$$

$$K = 0.106$$

If these values are substituted in Eq. (45) the value of $|V|$ is found to 0.9 per unit which checks the value shown in Fig. 27. Theoretically a modification of Eq. (45) should make it possible to determine the upper and lower value of the bus voltage $|V|$ during its oscillations when synchronism...
is lost. When the alternator with the open field is driven above synchronous speed the armature mmf acts alternately along the direct axis and along the quadrature axis of the field poles. Hence, for the maximum value of \(|V|\) the value of \(X_{du}\) should be in place of \((X_{du} + X_q)/2\) and for the minimum value of \(|V|\) the value of \(X_q\) should be substituted for \((X_{du} + X_q)/2\) and we have

\[
|V| \text{ max} = \frac{|V_o| (n-l)}{K \left( \frac{1}{X_{du}} + \frac{n-l}{K} \frac{\sin \theta_r}{n |Z_L|} \right)} \quad (46)
\]

\[
|V| \text{ min} = \frac{|V_o| (n-l)}{K \left( \frac{1}{X_q} + \frac{n-l}{K} \frac{\sin \theta_r}{n |Z_L|} \right)} \quad (47)
\]

If these values are substituted for the conditions obtaining for Fig. 28 we get

\[
|V| \text{ max} = 0.90 \text{ p.u.}
\]

\[
|V| \text{ min} = 0.88 \text{ p.u.}
\]

The value of \(|V| \text{ max}\) compares with the value of 0.90 obtained from Fig. 28. However, the same figure yields a value of 0.64 per unit for \(|V| \text{ min}\). It is possible that when the armature mmf lies in the quadrature axis that the tips of the shoes of the field poles saturate as under this condition one half of the total flux per pole is driven across the axis of the field pole. The pole shoes where they project beyond the pole core offer a relatively small area to the magnetic flux. The effect of saturation reduces the value of \(X_q\).

b) **Performance with an Open Exciter Field**

The effect of an open exciter field is similar to that of an open alternator field except that with the open exciter field there is a small
amount of current in the alternator field caused by the residual magnetism in the exciter. This means that the maximum real power which an alternator can deliver with an open exciter field is slightly greater than for an open alternator field, since $E$ in the first term of equation (38) is not zero. Oscillograms showing the effect opening the exciter field of one alternator of a two-machine system are included in Fig. 30 and Fig. 31.

The data for these conditions are listed in Table XXIII.

Table XXIII
LOAD ON ALTERNATOR D BEFORE OPENING EXCITER FIELD
(Name Plate Data Listed in Appendix III-A)

<table>
<thead>
<tr>
<th></th>
<th>Fig. 30 Synchronism Maintained</th>
<th>Fig. 31 Synchronism Lost</th>
</tr>
</thead>
<tbody>
<tr>
<td>KVA</td>
<td>19.4 (0.485 p.u.)</td>
<td>31.6 (0.79 p.u.)</td>
</tr>
<tr>
<td>KW</td>
<td>14.4 (0.36 p.u.)</td>
<td>23.3 (0.583 p.u.)</td>
</tr>
<tr>
<td>KVARs</td>
<td>13.0 (0.325 p.u.)</td>
<td>21.0 (0.525 p.u.)</td>
</tr>
<tr>
<td>P.F.</td>
<td>0.74</td>
<td>0.74</td>
</tr>
</tbody>
</table>

For the conditions obtaining for Fig. 30, the system voltage was reduced to 0.85 per unit and for Fig. 31 the voltage fluctuated between 0.62 and 0.82 per unit. On the basis of these voltages the real power load dropped from 0.36 per unit to $0.36 \times (0.85)^2 = 0.26$ per unit after the exciter field was opened for the case of Fig. 30. Similarly the real power load fluctuated between 0.39 per unit and 0.22 per unit for the case of Fig. 31. These data indicate that synchronism can be maintained for real power loads up to about 0.26 per unit when one of the two alternators operates with an open exciter field. This limit exceeds that for operation of an open alternator field by roughly 50 percent.
Fig. 30  Effect of Opening Exciter Field of One of Two Alternators
In Parallel. System Load 38.8 KVA, 0.74 p.f.
Fig. 31 Effect of opening exciter field of one of two alternators in parallel. System load 63.2 kVA, 0.74 p.f.
After synchronism is lost, the action is similar to that in the case of an open alternator field. However, the slip speed is smaller when the exciter field is opened than when the alternator field is opened, because in the case of the open exciter field, current is induced in the field circuit of the alternator which is closed through the armature of the exciter.

When an alternator loses its field excitation when operating in parallel with other alternators, it draws reactive power from the system for its excitation. If the effects of residual magnetism in the exciter are neglected, the reactive power drawn from the system when there is no field excitation is expressed by

\[
Q = \frac{|V|^2}{2X_d X_q} (X_d + X_q - (X_d - X_q) \cos 2\delta)
\]

(48)

The limiting value of \( Q \) would be

\[
Q_{\text{max}} = \frac{|V|^2}{X_q} = \frac{|V|^2}{0.9} = 1.10 |V|^2
\]

(49)

\[
Q_{\text{min}} = \frac{|V|^2}{X_d} = \frac{|V|^2}{1.5} = 0.67 |V|^2
\]

(50)

In Fig. 27, \( |V| = 0.9 \) per unit for that value of voltage the reactive power during loss of field excitation will have some value between 0.89 per unit and 0.54 per unit. The reactive power indicated in Fig. 27 is 0.53 per unit.

In view of this large drain of reactive power and the reduction in system voltage when an alternator loses its field excitation, and the field excitation cannot be immediately restored, that alternator should be disconnected from the system immediately.

b. Open Circuit in Reactive Current Equalizer Connection

If the equalizer connection between any two current transformers
shown in Fig. 19 is opened, all the secondary current in each current transformer will flow in the primary of the corresponding mutual reactor. If the system load is reactive then reactive current flows in each of the mutual reactors. The reactive regulating circuit in each alternator is affected as though each alternator delivers more than its share of reactive load. The result is reduced excitation and reduced system voltage. The greater the value of the reactive amplification factor $K$ the greater will be the reduction in system voltage in case the reactive equalizer circuit is opened.

c. Open Circuit in Current Transformer Connection

If a connection between the current transformer secondary and the primary of the mutual reactor should become open or if the secondary winding of the current transformer itself becomes open, current will result in the primaries of the mutual reactors. If there are $n$ alternators in parallel feeding a reactive load, the current in the primary of the mutual reactor associated with the open circuited current transformer is $I(n-1)/cn$ where $I$ is the current in each of the other current transformer primaries. The current in the primary of each of the other mutual reactors is $I/cn$ and its reactive component is such that the reactive current equalizer circuit causes a reduction in excitation in the corresponding alternators. The direction of the current in the primary of the remaining mutual reactor is opposite and causes an increase in the excitation of its alternator. Equilibrium is reached when the alternator with the open circuited current transformer carries the total reactive load as under these conditions there is no reactive component of current in any of the mutual reactors. This situation could cause the alternator, with the open circuited current transformer, to become overloaded.
d. Failure of Tachometer or Tachometer Circuit

If the tachometer or its circuit should fail in such a manner that the voltage to the pilot valve control goes to zero, the governor would increase the output speed of the constant speed drive. The speed of all alternators would increase in synchronism, for at least light load conditions. The real power equalizing circuit in the healthy units would indicate a deficiency of real power and cause their prime mover inputs to increase. The constant speed drives would increase their output speeds, one of them because the tachometer voltage is zero and the others because of the action of the real power equalizer circuit. It may be possible, at least under light load conditions, for the over-speed trip on any of the units to operate. In order to determine the exact behavior of the system under this condition, additional analytical and experimental studies should be made.

e. Short Circuit of Frequency Droop Circuit

From Fig. 18 it is evident that a short circuit of the frequency droop circuit of one alternator operating in parallel with others will reduce the output voltage, for example $E_{01}$, to zero. This will cause the droop circuits of the other alternators to circulate current in such a direction that real power load is shifted to the faulty alternator and the system frequency becomes lowered.

f. Failure of Potential Transformer in Frequency Droop Circuit

This effect is quite similar to that of a short circuit in the frequency droop circuit. In order to determine the exact behavior of the system when the frequency droop circuit fails, more test data are required. Future work should include experimental studies from which such data can be obtained.
g. **Summary**

If the excitation of an alternator operating in parallel with others becomes abnormal, the excitation of the other alternators will become abnormal in the same direction as a result of the action of the reactive power equalizing circuit. Thus if the voltage regulator of an alternator loses its sense of voltage, the exciter on that alternator will raise its excitation to ceiling value, and as a result this alternator will deliver more than its share of reactive power. Consequently the effect of the reactive load equalizing circuit will be such as to indicate a deficiency in the reactive power output in the other alternators, thus causing their exciters to increase excitation. As a result the bus voltage is sustained at a value above normal.

By the same process, loss of excitation in one alternator will produce a reduction in the excitation of the other alternators, thus causing the bus voltage to be sustained at a value below normal.

In the 2-alternator system, the voltage may rise to 13 percent above normal on over-excitation and decrease as low as 20 percent below normal on underexcitation if synchronism is maintained. Loss of synchronism produces a greater reduction of bus voltage.

Under conditions of abnormal excitation, the faulty alternator should be disconnected from the system.

Analytical and experimental work should be directed toward obtaining precise information governing the behavior of the electrical system during malfunctioning of the real power equalizing circuit and the speed regulating apparatus.
D. **Short-Circuit Currents**

Short-circuit currents were calculated for different types of faults using the alternator parameters presented in Section III-B. In the calculations, the transient impedances rather than the steady-state impedances are used and therefore the calculated results give values for the short-circuit currents immediately after the fault. The voltage regulator does not immediately act to increase the excitation of the alternator, and therefore its effect need not be considered for calculations of initial short-circuit currents.

The calculated results are compared with oscillographic records of single-line-to-ground and line-to-line faults in Table XXIV. Since data from oscillograms of a three-phase short circuit were used in determining the positive-sequence impedance as discussed in Section III-B, the calculated value of the three-phase fault current is not listed.

The short-circuit tests were made in the laboratory at Wright Field by applying solid faults to the bus, alternator A operating without load. Calculated and experimental values of short-circuit current are shown in Table XXIV. The oscillograms of the short-circuit currents from which the experimental values were evaluated are shown in Figs. 32, 33, 34, 35, 36, 37, 8, and 9.

The experimental and calculated results are in fair agreement. Some differences between the experimental and calculated results may be expected as some of the parameters upon which the calculations are based vary with magnetic saturation. There is also the difficulty of determining the exact value of the initial short-circuit current because the current decreases with time. The accuracy by which the magnitudes can be obtained from
## Table XXIV

**SHORT-CIRCUIT CURRENTS (ALTERNATOR-A)**

(Name Plate Data Listed in Appendix III-A)

<table>
<thead>
<tr>
<th>Type of Fault</th>
<th>Figure Number</th>
<th>$I_a$ (Amp)</th>
<th>$I_b$ (Amp)</th>
<th>$I_c$ (Amp)</th>
<th>Average (Amp RMS)</th>
<th>Calculated (Amp RMS)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Single-Line to-Ground</td>
<td>32</td>
<td>632</td>
<td></td>
<td></td>
<td>631</td>
<td>673</td>
</tr>
<tr>
<td></td>
<td>33</td>
<td></td>
<td>549</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>34</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Line-to-Line</td>
<td>35</td>
<td>494</td>
<td></td>
<td></td>
<td>559</td>
<td>455</td>
</tr>
<tr>
<td></td>
<td>36</td>
<td></td>
<td>549</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>37</td>
<td>576</td>
<td></td>
<td>549</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Three-Phase</td>
<td>8</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>713</td>
</tr>
<tr>
<td></td>
<td>9</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The values of short-circuit current were calculated using the following values of impedance.

- Transient Impedance of Alternator = $0.024 + j0.145$ P.U.
- Positive-Sequence Impedance of Line* = $0.019 + j0.005$ P.U.

$$Z_1 = 0.013 + j0.150$$ P.U.

- Negative-Sequence Impedance of Alternator = $0.137 + j0.217$ P.U.
- Negative-Sequence Impedance of Line* = $0.019 + j0.005$ P.U.

$$Z_2 = 0.156 + j0.222$$ P.U.

- Zero-Sequence Impedance of Alternator = $0.024 + j0.043$ P.U.
- Zero-Sequence Impedance of Line* = $0.021 + j0.011$ P.U.

$$Z_o = 0.045 + j0.057$$ P.U.

*The line simulated the impedance of 45 feet of $1$-number 12 wires per phase as given in Air Force Report, "A-C System Fuse Coordination Study" (December 1945), pp. 23.*
Fig. 33  B-Phase, Single Line-to-Ground Fault On One Alternator
Fig. 35  B-C Phases, Line-to-Line Fault On One Alternator
the oscillograms is somewhat limited.

The oscillograms in Fig. 32 to 37 inclusive show a strong double frequency component in the field current and a pronounced third harmonic component in the voltages of the unfaulted phases. The causes of these harmonics are discussed in Section III-A.

The values of short-circuit current shown in Table XXIV are symmetrical rms values. In addition, the d-c component must be considered for fast circuit breaker or fuse protective drives. The time constant of the d-c transient was found to be approximately 1 cycle (0.0025 sec.). It is probable that operating times as short as 1/4 cycle will be significant to fuse application. In 1/4 cycle the d-c component will decay to 0.7 of its initial value. Hence for completely offset waves the values of short-circuit current listed in Table XXIV should be multiplied by \( \sqrt{1^2 + (\sqrt{2} \times 0.7)^2} = 1.4 \).

E. Blocked Rotor Tests

For some types of mechanical failure, the rotor of an alternator may become blocked (stalled) while the alternator is operating in parallel with others. In order to obtain data for this condition, tests were made with the rotor of one 40-KVA alternator held in a blocked position while being fed by a similar alternator which was separately excited. The blocked rotor data for each phase are shown in Tables XXV and XXVI.

The data listed in Table XXV were obtained for three different standstill positions of the rotor of alternator I. Alternator I was selected because its rotor shaft was accessible. Alternator D was used as the source of power. Figures 38, 39, and 40 are vector diagrams corresponding to the three rotor positions.
Table XXV

BLOCKED ROTOR DATA FOR THREE POSITIONS - ALTERNATOR J

(Name Plate Data Listed in Appendix III-A)

<table>
<thead>
<tr>
<th>Rotor Position</th>
<th>( E_a ) Volts</th>
<th>( I_a ) Amps</th>
<th>( P_a ) Watts</th>
<th>( Q_a ) Vars</th>
<th>( \theta_a ) Degrees</th>
<th>( R_a ) Ohms</th>
<th>( X_a ) Ohms</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>20</td>
<td>105</td>
<td>2020</td>
<td>578</td>
<td>16</td>
<td>.183</td>
<td>0.0525</td>
</tr>
<tr>
<td>2</td>
<td>15.3</td>
<td>119</td>
<td>940</td>
<td>1555</td>
<td>58.9</td>
<td>.0665</td>
<td>0.110</td>
</tr>
<tr>
<td>3</td>
<td>33</td>
<td>57</td>
<td>1360</td>
<td>1260</td>
<td>42.8</td>
<td>.418</td>
<td>0.387</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Rotor Position</th>
<th>( E_b ) Volts</th>
<th>( I_b ) Amps</th>
<th>( P_b ) Watts</th>
<th>( Q_b ) Vars</th>
<th>( \theta_b ) Degrees</th>
<th>( R_b ) Ohms</th>
<th>( X_b ) Ohms</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>24.9</td>
<td>94.5</td>
<td>270</td>
<td>2330</td>
<td>83.4</td>
<td>.0302</td>
<td>0.263</td>
</tr>
<tr>
<td>2</td>
<td>33.5</td>
<td>68</td>
<td>-110</td>
<td>2250</td>
<td>92.8</td>
<td>-.0238</td>
<td>0.487</td>
</tr>
<tr>
<td>3</td>
<td>24</td>
<td>96</td>
<td>2060</td>
<td>952</td>
<td>26.7</td>
<td>.223</td>
<td>0.100</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Rotor Position</th>
<th>( E_c ) Volts</th>
<th>( I_c ) Amps</th>
<th>( P_c ) Watts</th>
<th>( Q_c ) Vars</th>
<th>( \theta_c ) Degrees</th>
<th>( R_c ) Ohms</th>
<th>( X_c ) Ohms</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>41.5</td>
<td>59</td>
<td>90</td>
<td>207</td>
<td>68.4</td>
<td>.0259</td>
<td>0.0655</td>
</tr>
<tr>
<td>2</td>
<td>36</td>
<td>74.4</td>
<td>1680</td>
<td>2100</td>
<td>51.3</td>
<td>.304</td>
<td>0.379</td>
</tr>
<tr>
<td>3</td>
<td>31</td>
<td>104</td>
<td>-1090</td>
<td>3110</td>
<td>109.3</td>
<td>-.099</td>
<td>0.283</td>
</tr>
</tbody>
</table>

Note: The alternator with the blocked rotor is treated as a load. (When the machine is receiving real and reactive power, \( P \) and \( Q \) are positive).
Table XXVI

BLOCKED ROTOR DATA FOR TWO EXCITATIONS - ALTERNATOR I

Rotor in One Position, But Using Two Value of Test Voltage.

<table>
<thead>
<tr>
<th>Phase a</th>
<th>Bus Voltage</th>
<th>$E_a$</th>
<th>$I_a$</th>
<th>$P_a$</th>
<th>$Q_a$</th>
<th>$\theta_a$</th>
<th>$R_a$</th>
<th>$X_a$</th>
</tr>
</thead>
<tbody>
<tr>
<td>on Open Circuit Volts</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>120</td>
<td>13</td>
<td>98</td>
<td>820</td>
<td>980</td>
<td>50</td>
<td>0.085</td>
<td>0.102</td>
<td></td>
</tr>
<tr>
<td>144</td>
<td>23.5</td>
<td>186</td>
<td>2320</td>
<td>3690</td>
<td>57.8</td>
<td>0.067</td>
<td>0.107</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Phase b</th>
<th>Bus Voltage</th>
<th>$E_b$</th>
<th>$I_b$</th>
<th>$P_b$</th>
<th>$Q_b$</th>
<th>$\theta_b$</th>
<th>$R_b$</th>
<th>$X_b$</th>
</tr>
</thead>
<tbody>
<tr>
<td>on Open Circuit Volts</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>120</td>
<td>28</td>
<td>64</td>
<td>-200</td>
<td>1785</td>
<td>96.4</td>
<td>-.049</td>
<td>0.435</td>
<td></td>
</tr>
<tr>
<td>144</td>
<td>53</td>
<td>103</td>
<td>-200</td>
<td>4980</td>
<td>92.3</td>
<td>-.019</td>
<td>0.470</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Phase c</th>
<th>Bus Voltage</th>
<th>$E_c$</th>
<th>$I_c$</th>
<th>$P_c$</th>
<th>$Q_c$</th>
<th>$\theta_c$</th>
<th>$R_c$</th>
<th>$X_c$</th>
</tr>
</thead>
<tbody>
<tr>
<td>on Open Circuit Volts</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>120</td>
<td>31</td>
<td>60</td>
<td>1000</td>
<td>1560</td>
<td>57.4</td>
<td>0.278</td>
<td>0.434</td>
<td></td>
</tr>
<tr>
<td>144</td>
<td>50</td>
<td>122</td>
<td>3720</td>
<td>4850</td>
<td>52.4</td>
<td>0.249</td>
<td>0.325</td>
<td></td>
</tr>
</tbody>
</table>
FIG. 38 - VECTOR DIAGRAM FOR ROTOR POSITION 1

SCALE:
1 INCH = 10 VOLTS
1 INCH = 40 AMPERES
SCALE:
1 INCH = 10 VOLTS
1 INCH = 40 AMPERES

FIG. 39 - VECTOR DIAGRAM FOR ROTOR POSITION 2
FIG. 40 - VECTOR DIAGRAM FOR ROTOR POSITION 3

SCALE:
1 INCH = 10 VOLTS
1 INCH = 40 AMPERES
Table XXVI lists data obtained with the rotor of alternator I in one standstill position but with the excitation of alternator D adjusted for an open circuit voltage of 120 volts and for an open circuit voltage of 144 volts.

Under the actual operating conditions for a 2-alternator system when the rotor of one becomes blocked, the excitation of the other will go to ceiling value. At that value of excitation 158 volts on open circuit, the value of the saturated synchronous reactance will probably be considerably lower than that corresponding to the excitation which produces 144 volts on open circuit. Therefore, values of current can be expected which are greater than the ratio 158/144 times the values listed in Table XXVI for an open circuit bus voltage of 144 volts. In a system with more than two alternators the bus voltage would suffer less reduction and the blocked rotor currents would be higher than in the system of two alternators.

Negative values of real power are shown in both Tables XXV and XXVI. This apparent negative real power is caused by the unequal mutual reactances between the phases of the alternator with its rotor at standstill. This unbalance is due to the uneven air gap resulting from the saliency of the rotor.

F. Synchronizing Disturbances

In order to connect a three-phase alternator to a live three-phase bus without causing fluctuations of bus voltage and power swings between the incoming alternator and those on the bus, the following conditions must be satisfied:

1. The phase sequence must be correct.

2. The voltage of the incoming alternator must be in phase with that of the bus.
3. The frequency of the incoming alternator must equal that of the bus.

4. The magnitude of the voltage of the incoming alternator must equal that of the bus.

The correct phase sequence of the alternator is assured at the time of its installation. In the case of large alternators such as are used in the central station industry, a phase angle meter known as a synchroscope is used to indicate the phase angle between the incoming alternator and the bus. For smaller alternators, such as are used in present aircraft installations, synchronizing lamps are used between the bus and the incoming alternator.

As the frequency of the incoming alternator approaches that of the bus the lamps flash on and off at a slow rate and the armature circuit breaker should be closed when the lamps are dark since this indicates that the voltages are in phase. Unfortunately because of the stress of an emergency or negligence on the part of the operator an alternator may be connected to the bus when its voltage is out of phase with that of the bus. If this phase displacement is large, severe fluctuations in the system voltage and large power swings between the incoming alternator and the system result.

Consider an unloaded bus supplied by (n-1) alternators and to which an alternator (alternator 1) is connected when its voltage leads that of the bus by the angle $\delta$. Figure 41 shows a diagram of the equivalent circuit of the bus and the incoming alternator, and also a vector diagram which shows the voltage, $E_1$, of the incoming alternator to lead the bus voltage $E_n$ by the angle $\delta$.

Let

$$E_n = |E| + j0$$

$$E_1 = |E| (\cos \delta + j \sin \delta)$$
a - Voltage relationship

\[ \delta \]

\[ E \]

\[ E_n \]

b - Equivalent circuit

\[ \frac{Z_d}{n-1} \]

\[ Z_d \]

\[ E_n \]

\[ E_1 \]

FIG. 41 - CONDITIONS AT TIME OF SYNCHRONIZING
If the effect of saliency is neglected the current is expressed by

$$I = \frac{|E|}{Z_d} \left( 1 - \cos \delta - j \sin \delta \right)$$

(51)

where:

- $n$ = total number of alternators including incoming alternator on the system
- $Z_d = R + j X_d'$
- $R$ = the resistance of the armature plus that of the leads to the bus
- $X_d'$ = the transient reactance of the alternator plus the reactance of the leads to the bus.

The terminal voltage, $V$, in terms of $E$ and $\delta$ is

$$V = E_n - \frac{Z_d}{n-1} I = |E| \left[ 1 - \frac{1 - \cos \delta}{n} + j \frac{\sin \delta}{n} \right]$$

(52)

The expressions for real and reactive power are respectively

$$P_1 = - (n-1) \frac{|E|^2}{n |Z|^2} \left[ R \left( \frac{n-2}{n} \right) (1 - \cos \delta) - X_d' \sin \delta \right]$$

(53)

$$Q_1 = - (n-1) \frac{|E|^2}{n |Z|^2} \left[ X_d' \left( \frac{n-2}{n} \right) (1 - \cos \delta) + R \sin \delta \right]$$

(54)

If values of $\delta$ are substituted into the power expressions with typical values of resistance and reactance, and the results plotted on rectangular coordinates as in Fig. 42, with real power as the abscissa and reactive power as the ordinate, it can be seen that two families of ellipses result. For a given value of $n$ there are two ellipses which are the loci for the power vector. The dotted ellipses represent the loci of the power vector for each of the machines of the bus and the solid ellipses are the loci for the incoming machine.

For a two machine system ($n=2$), the two ellipses become one.
FIG. 42—POWER RELATIONSHIPS ON SYNCHRONIZING
straight line through the origin. For the case where a machine is synchron-
ized to an infinite bus \((n=\infty)\) the ellipse for the incoming alternator be-
comes a circle and the ellipse for any one of the other machines degenerates
to a single point at the origin. This means that for the infinite-machine
case none of the machines already on the bus are affected by the incoming
machine, whereas the largest flows of reactive power to, and real power to
and from the incoming machine, occur.

For a system composed of three or more machines it can be seen
that a very small amount of reactive power is received by any one of the
machines originally on the system, just as the reactive power delivered by
the incoming machine is also very small. Real power is both delivered and
received by the incoming machine and all the other machines. The incoming
machine receives comparatively large amounts of reactive power.

Figure 43 shows oscillograms of field current, armature current,
voltage and reactive power when alternators A and D at Wright Field are syn-
chronized out of phase. If \(n=2\) is substituted in Eq. (53) and Eq. (54) the
real power and reactive power for the incoming alternator are expressed by
Eq. (55) and Eq. (56) below:

\[
P_1 = \frac{\frac{|E|}{2}^2}{2 |Z_d|^2} X_d' \sin \delta \\
Q_1 = -\frac{\frac{|E|}{2}^2}{2 |Z_d|^2} R \sin \delta
\]

The equations for the alternator which was originally on the bus are

\[
P_2 = -\frac{\frac{|E|}{2}^2}{2 |Z_d|^2} X_d' \sin \delta \\
Q_2 = \frac{\frac{|E|}{2}^2}{2 |Z_d|^2} R \sin \delta
\]
Fig. 43  Effect of Synchronizing Two Alternators
Out of Phase
From these equations it is evident that the ratio of P to Q is $\frac{X_{d}'}{R}$ for the 2-alternator system. In this analysis transient reactance $X_{d}'$ was chosen as that parameter which would give a good degree of approximation. The curves shown in Fig. 42 are based on the following values:

$$
\begin{align*}
R &= 0.043 \text{ p.u.} \\
X_{d}' &= 0.150 \text{ p.u.} \\
Z_{d} &= 0.156 \text{ p.u.}
\end{align*}
$$

The ratio of

$$
\frac{X_{d}'}{R} = \frac{0.150}{0.043} = 3.5
$$

From the oscillograms of Fig. 43 the ratio of $\frac{P}{Q}$ is found to be 3.7. These ratios are in good agreement. The values of power in Eq. (55) and Eq. (56) are the initial values which oscillate and gradually die out. Eq. (52) for the bus voltage shows that the power oscillations are accompanied by oscillations in the bus voltage. The most severe case is that of synchronizing 180 degrees out of phase i.e. $\delta = 180^\circ$. Under this condition the bus voltage goes to zero and the armature current from Eq. (51) is 6.5 per unit for the 2-alternator system. For an infinite system the bus voltage is unaffected but the current is 13.0 per unit or twice the available three-phase short-circuit current. The effects of reduced voltage and high circulating currents should be investigated to determine their effects on load equipment and protective devices.

Such an extreme transient current as 13.0 per unit would probably cause the exciter to reverse its polarity. Because of the large rise in alternator field current, the resistance drop in the exciter armature circuit could well be greater than the voltage induced in the exciter armature.

This would cause the terminal voltage of the exciter to reverse, which in
turn would produce a reversal of exciter field current. The voltage regulator will not function properly with reverse exciter field because the action of the stabilizing transformer TR3 in Fig. 13 will be in the wrong direction. Normal operation of the voltage regulator can be restored by reversing the polarity of the stabilizing transformer. It may be better to prevent the reversal of polarity by use of an appropriate series field on the exciter. A series field may also be advantageous by preventing polarity reversals on short circuits and by increasing the exciter response.\(^1\)

**G. Overvoltage**

A-C electric power systems are subject to two general kinds of overvoltage. The first of these is caused by overexcitation of the alternators on the system. This overvoltage is less than twice rated voltage and is sustained for relatively long periods of time at system frequency. The other kind of overvoltage is transient and on a 3-phase, 208 volt, 400 cycle system; may have values of several thousand volts for very short durations. These may result when current is interrupted at other than zero value and are due to the inductance of the system.

1. **Overvoltage Due to Voltage Regulator Failure**

If the voltage regulator on one of two alternators operating in parallel fails, so as to drive the excitation of that alternator to ceiling value, the bus voltage will attain a maximum sustained value of 136 volts line-to-neutral as shown in Section III-C-4. If there is reactive load on the system the overvoltage is not as great. Figure 44 shows an oscillogram

---

Fig. 44. Ceiling Excitation on One of Two Alternators in Parallel
of the effect of regulator failure on one of two alternators when there is no load on the bus. The excitation reaches ceiling value 0.04 seconds after the regulator fails. After a period of about 2 seconds the exciter protection relay operates and causes the alternator to be disconnected from the system. The overvoltage on the bus in the case of one alternator attaining ceiling excitation is determined by the number of alternators on the system, the system load, and the strength of the reactive load equalizer. The greater the number of alternators on the system, the smaller will be the overvoltage. The stronger the reactive load equalizer, the greater will be the overvoltage.

Under light load conditions with only one alternator connected to the bus, ceiling excitation would produce a line-to-neutral voltage in the order of 158 volts. Even on a multiple alternator system, there may be only one alternator connected to the bus at a given time. Therefore the single alternator case must always be considered for determining possible overvoltage.

2. 400-Cycle Overvoltage Due to Load Switching and Fault Clearing

If a short circuit persists for a period greater than 0.03 seconds, the excitation of the alternators increases and approaches ceiling value. Removal of the fault will give rise to an overvoltage. A similar situation is created when a heavy load is switched off. Should the entire load be suddenly removed, the terminal voltage of each alternator would rise to the open circuit voltage corresponding to its excitation. This will result in a high bus voltage that will last until the voltage regulator restores it to normal. The removal of a short circuit defines the maximum overvoltage that would result on heavy load switching.
A test was run where alternator A was connected to the bus, and while it was running as an unloaded machine, a three-phase short circuit was applied. Figure 9, an oscillogram of a three-phase fault at the bus, shows that ceiling excitation is reached 0.07 seconds after application of the fault. The alternator field current remains constant at the ceiling value. The fault was cleared after 0.54 seconds and produced the overvoltage shown in the oscillogram of Fig. 45. For the situation represented by this oscillogram, after the fault was removed 0.07 seconds were required before the voltage regulator started to reduce the alternator field current.

To evaluate the heating effect due to the overvoltage, the envelope of phase a voltage was sketched on the oscillogram in Fig. 45. The instantaneous voltages at the various times after the fault is cleared were obtained from the oscillogram and are shown in Table XXVII. Since the heat produced in a constant resistance is proportional to the voltage squared, the energy during a time interval is proportional to the square of the voltage multiplied by the time. The total accumulated energy at any time after the fault is removed is obtained by summing the energy over each time interval up to that time. The equivalent constant voltages that would also produce the same accumulated energy when maintained for different durations are also calculated in Table XXVII, and their effective values are plotted in Fig. 46. An interpretation of the curve means that after 0.1 seconds the amount of heat produced by the type of overvoltage whose envelope is indicated on the oscillogram is the same as that produced by a constant voltage whose effective value is 155.5 volts line-to-neutral and is applied for a period of 0.10 second.

These data help to define a portion of the general voltage-time
Fig. 45  Removal of a Three-Phase Short Circuit From One Alternator
### Table XXVII

OVERVOLTAGE CALCULATIONS
ON REMOVAL OF A THREE-PHASE SHORT CIRCUIT

<table>
<thead>
<tr>
<th>Time in Sec. ( t )</th>
<th>Instantaneous Voltage ( e )</th>
<th>Proportional to Energy ( (e^2)(\Delta t) )</th>
<th>Accumulated Energy ( W )</th>
<th>Square of Max. Volt. ( W/t )</th>
<th>Effective Voltage</th>
</tr>
</thead>
<tbody>
<tr>
<td>.005</td>
<td>170</td>
<td>288</td>
<td>288</td>
<td>28800</td>
<td>120</td>
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<tr>
<td>.01</td>
<td>217</td>
<td>471</td>
<td>759</td>
<td>37950</td>
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<td>511</td>
<td>1270</td>
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<td>44500</td>
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<td>46700</td>
<td>153</td>
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<tr>
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<td>226</td>
<td>511</td>
<td>3314</td>
<td>47300</td>
<td>153.5</td>
</tr>
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<td>511</td>
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<td>48200</td>
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</tr>
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<td>48500</td>
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<td>48500</td>
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<td>49200</td>
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<td>46800</td>
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<tr>
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<td>226</td>
<td>511</td>
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<td>31535</td>
<td>125</td>
</tr>
</tbody>
</table>

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FIG. 46 - OVERVOLTAGE CURVES
curve that equipment operating on the system must withstand. The particular curve shown in Fig. 46 is based on the removal of a three-phase short circuit after the alternator has reached ceiling excitation, but as previously stated, it also provides the upper limit for load switching. The overvoltages due to load switching would in most cases be lower since the excitation may not be at as high a value prior to the switching. The duration of the overvoltage could be reduced if the voltage regulator response were faster.

Overvoltages may occur on the non-faulted phases during a single-line-to-ground fault. This results from action of the voltage regulator to increase excitation in response to the reduced voltage on one phase of the V-V transformer. This voltage will not exceed the ceiling voltage of 158 volts line-to-neutral and will occur only if the fault is not cleared prior to regulator action. The voltage will persist until the fault is cleared.

3. Overvoltage Due to Errors in The Voltage Regulator

On the present 400 cycle a-c system the bus voltage is normally maintained at a value of 208 volts line-to-line through the action of the voltage regulator. The accuracy of this setting of 208 volts will depend upon both the human error of the operator in setting the voltage prior to synchronizing, and the accuracy of the instrument used for the setting. Even though the setting is accurately made, the bus voltage may drift. The specified allowable error in the voltage regulator is 2.5% of 208 volts (MIL-R-5292A) throughout the operating range of the alternator. Therefore, if the regulator setting on one alternator is at one value and the others at a different value or values, the resulting bus voltage will be some average of the various settings. All these factors may combine to produce a bus voltage that may be higher than normal.
4. **Transient Overvoltages**

Transient overvoltages last for only a short time, and are due to interrupting the current at a time other than when it is passing through zero. In this way a voltage proportional to \( \frac{di}{dt} \) is produced. This voltage persists for an extremely short time but may be of such magnitude that flashover and insulation puncture result.

The interruption of the current at a time other than zero occurs if current limiting fuses are used. With ordinary fuses, the current may also be interrupted other than when it is zero providing the current passing through the fuse is large enough (Section IV-B). Such large currents might flow during short circuits on a multi-alternator system.

The energy associated with these transient voltages is small and will not cause appreciable overheating. However, the magnitudes of such voltages may rise to several thousand volts and unless the components of the system and the utilization devices are insulated properly, dielectric breakdown will occur. These overvoltages should be investigated in future studies with a possible view toward finding methods for suppressing them or to determine what the insulation levels of aircraft electrical systems should be.
Name plate data of alternator D

Westinghouse Electric Corp.
Alternator, Engine Driven, Type A-1
40 KVA, 0.75 p.f., 208/120 v., 111 amps, 3φ, 400 cycles
6000 rpm, Spec. No. 32509
Ser. AF-47-296, Contract W33-038-AC-14194

Name plate data for constant speed drive of alternator D

Sundstrand Machine Tool Co.
Sundstrand, Drive Alternator Constant-Speed
Input 2800 to 9000 RPM
Output 6000 RPM, 50 HP
Part No. 49 HT-30h, Ser. No. 382
Spec. No. OST-2076
Manufactured for General Electric Co.

Name plate data of drive stand of alternator D

U.S. Varidrive Syncrogear
HP 50, Speed Range 1:1, Gear Ratio 6 to 21 incl.
Type VEU-GSTT, RPM Max-9000
RPM Min-2400, Ser. No. 497862
150/75 amp F.L., 1200 Motor RPM
3φ, 60 cycle, 220/440
U.S. Electric Motor, Inc.
Appendix -B

FREQUENCY DROOP CIRCUIT SENSITIVITY

The schematic diagram of the frequency droop circuit is shown in Fig. 17. The effect of inphase and quadrature current on the output voltage of the droop circuit is determined as follows:

In Fig. 17 the primary voltage of transformer $T_1$ is $V$ volts per phase. If the ratio of $T_1$ is $m$, the voltages across the two halves of the secondary $T_1$ are

\[ V_{34} = m \frac{V}{2} \]
\[ V_{45} = m \frac{V}{2} \]

where:

- $V_{34}$ is the voltage drop from terminal 3 to terminal 4 of transformer $T_1$
- $V_{45}$ is the voltage drop from terminal 4 to terminal 5 of transformer $T_1$

A current transformer, of ratio $c$, is in the same phase as $T_1$. If the line current is $I$, then the secondary current $\frac{I}{c}$ produces a drop across the resistor $R_1$. The voltage equations for the two meshes containing $R_1$ are:

\[ 3V_{12} = V_{45} + \frac{I}{c} R_1 \]  \hspace{1cm} (1)
\[ 2V_{12} = V_{34} - \frac{I}{c} R_1 \]  \hspace{1cm} (2)

where:

- $3V_{12}$ is the voltage drop from terminal 1 to terminal 2 in transformer $T_3$
- $2V_{12}$ is the voltage drop from terminal 1 to terminal 2 in transformer $T_2$

Let $I$ be divided into an inphase component, $I_p$, and a quadrature component,
The expression for $I$ in terms of $I_p$ and $I_q$ is:

$$I = I_p - j I_q$$  \hspace{1cm} (3)

$I_p$ is positive when real power flows from the alternator and $I_q$ is positive if reactive power flows from the alternator (current lagging).

Eqs. (1) and (2) are rewritten as

$$3V_{12} = (1V_{45} + \frac{I_p}{c} R_1) - j (\frac{I_q}{c} R_1)$$  \hspace{1cm} (4)

$$2V_{12} = (1V_{34} - \frac{I_p}{c} R_1) + j (\frac{I_q}{c} R_1)$$  \hspace{1cm} (5)

If the ratios of transformers $T_2$ and $T_3$ are each $t$, then the secondary voltages are:

$$3V_{34} = t \cdot (3V_{12}) = t (1V_{45} + \frac{I_p}{c} R_1) - j (\frac{I_q}{c} R_1)$$

$$2V_{12} = t \cdot (2V_{12}) = t (1V_{34} - \frac{I_p}{c} R_1) + j (\frac{I_q}{c} R_1)$$

The magnitudes are:

$$|3V_{34}| = t \sqrt{(1V_{45} + \frac{I_p}{c} R_1)^2 + (\frac{I_q}{c} R_1)^2}$$

$$|2V_{12}| = t \sqrt{(1V_{34} - \frac{I_p}{c} R_1)^2 + (\frac{I_q}{c} R_1)^2}$$

The d-c voltage outputs of the rectifiers SR1 and SR2, with the addition of the shunt smoothing capacitors C1 and C2 and the bleeder resistor $R_2$, are approximately equal to the effective values of the a-c voltages $2V_{34}$ and $3V_{34}$, respectively. The voltage across $R_2$ is the difference between the two rectified magnitudes of $3V_{34}$ and $2V_{34}$. If $E_o$ is the voltage across $R_2$ and the drop in the rectifiers is neglected then

$$E_o = |3V_{34}| - |2V_{34}|$$

$$E_o = t \sqrt{(1V_{45} + \frac{I_p}{c} R_1)^2 + (\frac{I_q}{c} R_1)^2} - \sqrt{(1V_{34} - \frac{I_p}{c} R_1)^2 + (\frac{I_q}{c} R_1)^2}$$

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or since \(1^{\frac{3}{4}} = \frac{mV}{2}\)
and \(1^{\frac{4}{5}} = \frac{mV}{2}\)

\[
E_o = t \left[ \sqrt{\left(\frac{mV}{2} + \frac{I_p}{c} R_1\right)^2 + \left(\frac{I_q}{c} R_1\right)^2} - \sqrt{\left(\frac{mV}{2} - \frac{I_p}{c} R_1\right)^2 + \left(\frac{I_q}{c} R_1\right)^2} \right]
\]

The constants for the droop circuit have the following values.

\[
c = 125, \quad t = \frac{21}{25} = .840
\]
\[
m = \frac{30}{208} = .144, \quad R_1 = 3 \text{ ohms}
\]

\[
E_o = .840 \left[ \sqrt{(.072 V + .024 I_p)^2 + (.024 I_q)^2} - \sqrt{(.072 V - .024 I_p)^2 + (.024 I_q)^2} \right]
\]

Fig. 22 shows the family of curves of \(E_o\) versus \(I_p\) which result from variation of the parameter \(I_q\). This family of curves shows that the reactive current, \(I_q\), has negligible effect upon the output voltage \(E_o\) and consequently on the droop characteristic within the normal current range.

For example, if two alternators are operating in parallel and one of these, alternator 1, goes to ceiling excitation and carries 0.8 per unit inphase component of current and 1.50 per unit reactive component of current the output voltage of the frequency droop circuit \(E_{o1}\) is 3.32 volts from Fig. 22. The action of the governors on the two alternators is such as to make the droop circuit output voltages equal i.e. \(E_{o1} = E_{o2}\). If the reactive current in alternator 2 is negligible the inphase current corresponding to a value of 3.32 volts for \(E_{o2}\) in Fig. 22 is 0.75 per unit. The inphase current in each alternator if divided equally would be 0.775 per unit. The error in the division of real current for an unbalance in reactive current of 1.5 per unit is 0.025 per unit.
IV. METHODS OF PROTECTION OF A-C SYSTEMS

A. General Protection Problems

1. Nature of Short Circuits

Reliable information on short circuits that occur in the field seldom can be obtained. The examples on which information is available emphasize the importance of adequate short circuit protection which will detect and clear the short circuit in the shortest possible time. The power system engineer should assume that a short circuit can occur in any part of the electric power system and should install protective equipment to minimize the consequences of the fault, should it occur.

Several series of short circuit tests have been carried out by a number of investigators in the laboratory. For each of these tests it has been necessary to artificially apply the short circuit. Various techniques of applying the fault have been used with the hope that the fault would be representative of those that occur in practice.

1 "A-C System Fuse Coordination Study", AAF Memorandum Report Serial No. TSEPE-656-669BV (4 December 1945).


Any of the above series of tests could be objected to on the basis that the particular method of applying the short circuit was not a typical fault. It is believed that such an objection might miss the significant point that although the short circuit may not be typical it certainly might occur. Adequate protective schemes should be capable of clearing that particular type of short circuit as well as other perhaps more representative types.

It is evident, however, that still additional short circuit investigations are desirable. These tests should attempt to explore the range of possible short circuit conditions by applying faults in many different manners. Such tests would permit more intelligent applications of protective devices to provide protection for the most extreme possible faults.

2. "Fail safe"

The term "fail safe" is not completely self-descriptive and is subject to some confusion in interpretation. Possible interpretations are:

(1) Protection failure trips circuit. Failure of the protective circuit results in opening of the protected circuit.

(2) Protection failure does not trip circuit. Failure of the protective circuit results in the protected circuit remaining in operation without protection.

The former would be considered unsafe if such a failure and consequent loss of the circuit will result in unsafe operation of the airplane.

The latter would be unsafe if a subsequent fault should occur on the unprotected circuit.

It should be noted that no system completely meets one or the other of the "fail safe" definitions. Each system can fail in several ways and tripping or non-tripping will depend on the nature of the failure. This factor will be illustrated in Section IV-D in the discussion on differential
protection. However, in most cases, some types of failure can be expected to be more likely than other failures. Hence it is probably possible to conclude whether failure of a protective scheme is more likely to meet one of the definitions than the other.

Circuit failures on a well installed power system are a relatively rare occurrence whether the failure is an open circuit or a short circuit. None-the-less since such failures may at some time occur alternate sources of supply to essential loads must be used. For example more than a single power source is desirable and should be adopted wherever at all possible. Similarly multiple channel feeder circuits will be used.

Such duplication must be made in order to provide continuity of power supply in spite of failure of the circuits for any cause. In the case of an alternator in particular there are many different devices and circuits whose failure will result in loss of the machine. Once the power supply system is properly designed to admit such a failure without loss of the airplane, tripping of the circuit on the rare occasion of failure of the protection will—except in still more rare circumstances—result in no hazard to the aircraft. Failure of the circuit or protective equipment will have been indicated by tripping the circuit and corrective maintenance will be assured before a subsequent flight.

Where protection failure will not trip the circuit breaker there will be hazard to the aircraft only if a short circuit subsequently occurs on the unprotected circuit. Although the dual circumstance of protection failure and circuit faults may be rare, it may be quite possible for the following reasons:

(1) In combat operation, the externally inflicted damage which may cause failure of the protective circuits may be extensive enough to cause
a fault on the protected circuit, and at least some points the circuits must be in close proximity.

(2) Failure of the protective circuit will not be indicated by tripping of the main circuit. It is doubtful if war-time maintenance conditions will result in the preventative maintenance checks necessary to indicate repair of the protective circuit immediately following the flight during which the failure occurred. It is more likely that the aircraft will continue in operation without adequate short circuit protection.

It is judged that for the normal aircraft where duplication of power supply service is possible, protection should be adopted of such nature that failure of the protection is most likely to trip the circuit. Systems of protection whose failure is likely to leave the circuit in operation without protection should be adopted only on those aircraft where weight is of extreme importance, where adequate duplication of service is not possible.

3. Types of Protection

Methods of detection of faults on a-c systems have been extensively considered for utility and industrial a-c systems. Because of complexity not all of these methods should be considered for aircraft systems. Some of those types of protection which appear applicable include:

(1) Overcurrent
(2) Directional power
   (a) Real
   (b) Reactive
(3) Differential current
(4) Balanced current
(5) Ground current

(6) Impedance or Reactance

Since the fundamentals of these methods are adequately described in the literature 1,2,3 no attempt will be made to cover them in detail in this report. Application of the above listed methods will be reviewed. Because of its relative importance, considerable work has been carried out on differential protection.

B. Overcurrent Protection

Overcurrent protection may involve either circuit breakers or fuses. The use of fuses on a-c distribution systems for aircraft has been extensively considered. The characteristics of the fuses now used are described in Specification MIL-F-5372(USAF) dated 23 October 1950. The melting characteristics of Fig. 1 of that report are reproduced here as Fig. 47 with the time scale extended into the area below 0.01 seconds assuring the $I^2t$ energy for melting is a constant for such short time.

Table XXVIII gives values determined from the fuse characteristics with melting current expressed as a multiple of the nominal current

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4"AC System Fuse Coordination Study", AAF Memorandum Report Serial No. TSEPB-656-669-BV (4 December 1945).
FIG. 47 - MELTING TIME - CURRENT CURVES FOR 400 CYCLE, SINGLE PHASE FUSES (MIL-F-5372(USAF))

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-153-
<table>
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<th>Melting Time (Sec.)</th>
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<th>20Amp</th>
<th>30Amp</th>
<th>40Amp</th>
<th>50Amp</th>
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<td>84</td>
<td>87</td>
<td>84</td>
<td>88</td>
</tr>
<tr>
<td>.01</td>
<td>32</td>
<td>29</td>
<td>27</td>
<td>27</td>
<td>26</td>
<td>28</td>
<td>26</td>
<td>28</td>
</tr>
<tr>
<td>.1</td>
<td>10</td>
<td>9.0</td>
<td>8.3</td>
<td>8.5</td>
<td>8.0</td>
<td>8.7</td>
<td>8.3</td>
<td>8.4</td>
</tr>
<tr>
<td>1.0</td>
<td>3.7</td>
<td>3.4</td>
<td>3.5</td>
<td>3.4</td>
<td>3.4</td>
<td>3.7</td>
<td>3.6</td>
<td>3.6</td>
</tr>
<tr>
<td>10.0</td>
<td>2.4</td>
<td>2.5</td>
<td>2.6</td>
<td>2.6</td>
<td>2.6</td>
<td>2.7</td>
<td>2.6</td>
<td>2.6</td>
</tr>
<tr>
<td>100</td>
<td>2.2</td>
<td>2.4</td>
<td>2.6</td>
<td>2.6</td>
<td>2.6</td>
<td>2.3</td>
<td>2.3</td>
<td>2.3</td>
</tr>
<tr>
<td>1000</td>
<td>2.2</td>
<td>2.4</td>
<td>2.3</td>
<td>2.3</td>
<td>2.2</td>
<td>2.3</td>
<td>2.3</td>
<td>2.2</td>
</tr>
</tbody>
</table>
rating. It will be noted that these values are very nearly the same for the full range of ratings and hence the fuse characteristics can be shown by a single curve. Figure 48 applies specifically to the 100 ampere rating but it is representative of all of the fuses.

The specified maximum clearing time and manufacturing tolerances of the fuses are shown in Fig. 49. The curves in this figure indicate that if the clearing time required by various fuses may range from extremely fast operation up to specified limit of 1/2 cycle, co-ordination between fuses in a fuse protected network might be difficult to obtain where high fault currents are possible.

Fuses are normally considered to have current limiting action. Some of this results from the resistance of the fuses itself. However, in true current limiting fuses, current is limited by the rapidity with which the fuse interrupts the circuit. That is the fuse operation is sufficiently rapid that it acts before the short-circuit current reaches a peak value. For 400-cycle a-c systems true current limiting action could occur only for a melting time of less than 1/4 cycle (0.6 milliseconds). As shown by the fuse characteristics this will require short-circuit currents in excess of 125 times nominal rating. Such currents are available in multigenerator systems.

It is important to note that current limiting fuse action may also induce high transient overvoltages because of the high rate of change of current as the short-circuit current is interrupted. (See Section III-G).

Current limiting fuses need not be capable of interrupting the actual maximum current that may occur in a system since they are effective in limiting the current to a lower value. As with other protective devices,
Fig. 48 - Typical fuse characteristic for 400 cycle, single phase fuses.
FIG. 49 - CHARACTERISTIC CURVES SHOWING MANUFACTURING AND CLEARING TOLERANCES OF 400 CYCLE, SINGLE PHASE FUSES
the interrupting rating should be expressed in terms of available short-
circuit currents. Available short-circuit current is that current which
would flow if the protective device were replaced by a bar of negligible
impedance. Specification MIL-F-5372 should be revised to define the fuse
rating in terms of available currents.

Because of the very fast action of fuses at high currents they may
interrupt the circuit before the d-c component has had time to decay. The
fuses therefore should be rated to interrupt an available current assuring
a fully off-set short-circuit current. If no decay of the d-c component
occurs during the interrupting time the assymmetrical rms current would be
1.73 times the symmetrical rms fault current. Since some decay does occur
a multiplying factor of 1.4 is suggested in Section III-D. Specification
MIL-F-5372 should be revised to designate the appropriate multiplying factor.

In order to check protective devices with offset d-c components,
methods for closing in faults at any desired point in the cycle should be
developed for laboratory use.

The fault current available on large a-c systems now in use and
contemplated may be sufficient to result in melting times of less than 0.01
seconds. Because of differences in interrupting time, co-ordination is
usually more difficult at the higher fault currents. Specification MIL-F-
5372 does not specify accuracy of melting characteristics at times of less than
0.01 seconds or at more than 50 times normal current. It is desirable to ex-
tend the requirements on melting characteristics and accuracies down to melting
times representative of the maximum available short-circuit current. For
4000 amp available current this would represent the following melting times.
Fuse Rating-Amp. | Per Unit Current | Melting Time-Sec.
---|---|---
10 | 400 | 0.00007
20 | 200 | 0.00023
30 | 133 | 0.00050
40 | 100 | 0.00085
50 | 80 | 0.0013
60 | 67 | 0.0018
80 | 50 | 0.0033
100 | 40 | 0.0050

It is doubtful if melting time of less than 1/4 cycle will be significant. Variations in interrupting time will probably prevent co-ordination at such low values of time even though accurate melting characteristics could be attained. Hence, the requirements should extend down to the values shown above for the larger ratings but not lower than 0.0006 second for the smaller rating.

C. Directional Protection

Directional protection has been studied only in connection with generator circuit protection. Its characteristics are discussed in Section V-A. However, directional current protection may also be applicable for feeder protection as discussed in Section V-B.

D. Differential Current Protection

Differential current protection involves a comparison of currents at the two terminals of a circuit. For generator or motor protection the currents of each phase are compared at the neutral and at the interconnection of the phase lead with its associated bus. This affords protection for both the generator or motor itself and for any of the cable circuit included in the zone between current comparison points—the protected zone. For line protection the currents at the two ends of the line are compared. For bus protection the current flowing into and out of the bus are compared.

The following discussion relates particularly to differential pro-
tection for generator and line circuits. Two systems of differential protection have been proposed for use on aircraft alternators:

1. Single current transformer arrangement—Fig. 50a
2. Two current transformer arrangement—Fig. 50b.

The former method makes use of one thru-type current transformer in each phase with the bus-end lead and neutral-end lead brought thru the window of the current transformer so that the net mmf applied to the core is zero when the currents in the two ends of the phase are the same. The secondary winding of the current transformer connects to a simple current relay as shown in Fig. 50a. The other system of differential protection is shown in Fig. 50b. In that arrangement one thru-type current transformer is connected into the bus-end and another identical current transformer connected into the neutral-end. The secondaries of the two current transformers are connected in series by means of pilot wires. A simple current relay is connected across the secondary of one of these current transformers. Connections are made with the polarities so that under normal conditions, i.e. the same primary current in each of the two current transformers flowing in the same direction, negligible current will flow in the relay.

1. Single Current Transformer Arrangement

The single-current transformer arrangement shown in Fig. 50a is generally used in installations in which the distance from the alternator to the bus is relatively short as the current transformer is usually placed as close to the bus as possible in order to protect the leads from the alternator to the bus as well as the armature of the alternator. For large distances between the alternator and the bus, the length and weight of neutral lead in the primary of the current transformer becomes excessive.
a - SINGLE CURRENT TRANSFORMER DIFFERENTIAL PROTECTIVE SCHEME. ONE CURRENT TRANSFORMER AND ONE RELAY FOR EACH PHASE.

b - TWO CURRENT TRANSFORMER DIFFERENTIAL PROTECTIVE SCHEME. TWO CURRENT TRANSFORMERS AND ONE RELAY FOR EACH PHASE.

C - CONSTRUCTION OF CURRENT TRANSFORMER CORE
The chief advantage of the single current transformer arrangement is its simplicity and its freedom from nuisance operations provided the secondary winding of the current transformer is distributed uniformly around the core. The current transformers tested in this investigation have a secondary winding of 27 turns occupying less than one-half the circumference of the core. The core is in the form of a hollow cylinder as shown in Fig. 50c.

The name plate data on the case which houses three such current transformers (Fig. 50b) and three relays follow:

Relay, Alternator Differential Current Protection
Order No. (33-038) 46-4251-P
Exhibit TSEPE-6E-37
Mfr. Part No. 10346 Ser. No.
Standard Electrical Products Co.
Dayton, Ohio

a. Tripping Checks

Tripping and thru fault checks were made. The tripping check was made by placing one primary turn thru the window of the current transformer, Fig. 51a. The following values of 400-cycle current caused relay operation.

<table>
<thead>
<tr>
<th>Current Transformer</th>
<th>Primary Amperes</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>25.9</td>
</tr>
<tr>
<td>B</td>
<td>35.0</td>
</tr>
<tr>
<td>C</td>
<td>27.9</td>
</tr>
</tbody>
</table>

The tripping check was repeated on current transformer A but with the secondary windings of current transformers B and C in parallel with the secondary winding of current transformer A as shown in Fig. 51b. The primary current in current transformer A to produce tripping of the relay was
**Fig. 51** - Circuit diagrams for tripping checks on differential protection.

**a** - Test circuit for tripping check. Secondaries of two current transformers in parallel with relay under test.

**b** - Test circuit for tripping check. Secondaries of two current transformers in parallel with relay under test. The relay is represented by R. The ammeter A is in the secondary of an instrument current transformer.
found to be 32.2 amp. The object of this test was to determine the sensitivity of the protection if only one relay is used in conjunction with three current transformers. Such an arrangement would not clear line-to-line faults if the ratios of the three current transformers are equal. However, line-to-ground faults would be cleared with somewhat lower sensitivity than in the three-relay scheme.

b. Thru-Fault Checks

In the thru-fault test, normal connections were simulated by bringing a lead and its return thru the window of the current transformer as shown schematically in Fig. 52a. Single-line-to-ground faults were applied thru this arrangement to one 400-cycle alternator and to two 400-cycle alternators in parallel. Fig. 53 shows oscillograms of primary current and secondary current for a thru fault of 760 amp a-c supplied by one alternator. The secondary current i.e. the relay current was 0.07 ampere rms. The relay did not trip. Fig. 54 shows oscillograms for a thru-fault current of 1520 amp a-c supplied by two alternators in parallel. The relay current was found to be 0.19 ampere rms which is insufficient to trip the relay. The relay tripping current is of the order of 1.0 ampere based on the tripping checks.

The relay current on thru fault checks is probably due to the unsymmetrical arrangement of the secondary winding. Because of this dissymmetry the coupling between the secondary winding and one of the primary conductors is different from the coupling with the other primary conductor. Although the relay current on thru fault seems to be well below the value required to trip the relay it could probably be eliminated almost entirely if the secondary winding were distributed uniformly around the circumference of the core of the current transformer. Such an arrangement would practically guar-
a - Test circuit for thru-fault check on single current transformer scheme

b - Test circuit for thru-fault check on two-current transformer scheme

The relay is represented by R
The ammeter A is in the secondary of an instrument current transformer

Fig. 52 - Circuit diagrams for thru-fault checks on differential protection.
Fig. 53 Through-Fault Check on Single C. T. Differential Scheme. $I_F = 760$ Amperes
Fig. 54 Through-Fault Check on Single C. T. Differential Scheme. $I_F = 1520$ Amperes
antee against nuisance operations on thru fault.

c. Summary

The differential scheme using only one current transformer has the following advantages:

(1) Simplicity
(2) Higher Sensitivity
(3) Freedom from nuisance operation without any sacrifice in sensitivity.

If the secondary wire becomes open circuited or short circuited the protected circuit is not tripped but continues in operation without differential protection.

The single transformer arrangement is applicable only where the neutral can readily and economically be available. Hence, differential current protection using the single transformer type will be applicable only to motor or generator protection.

2. Two-Current Transformer Arrangement

A scheme of differential current protection involving two-current transformers per phase is shown in Fig. 50b. This principle has been used in various forms for many years and is considered perhaps the most effective in the electric power industry. The single-current transformer scheme shown in Fig. 50a has not come into appreciable use for protection of large alternators such as are used in the electrical power industry for the following reasons:

(1) Great distance between the alternator and the station bus. In many installations the bus is in a different building than the alternator.

(2) Limitations in physical size of the current transformer. The primary conductors are large and a large amount of dielectric would be required.
to insulate the two primary conductors from each other.

Theoretically, the two-current transformer arrangement is somewhat less sensitive than the single-current transformer arrangement. It is evident that on an internal fault check (minimum trip current) the scheme of Fig. 50a would require somewhat less primary current than the value applied to the primary of only one of the two-current transformers in Fig. 50b as in the latter the secondary of its companion current transformer is in parallel with the relay R. This small difference in sensitivity, however, has no practical significance.

The two-current transformer arrangement requires some precautions to prevent tripping on thru faults if the relay is to be kept as small and as simple as possible. These precautions, unnecessary in the single-current transformer scheme, are as follows:

1. The two current transformers must have very nearly equal ratio and phase angle characteristics for a large range of current values. The current transformers must be matched. They must have the same number of secondary turns, the same core dimensions and the cores must be of the same magnetic material.

2. It is necessary to avoid large differences in the impedance of the leads from the secondary terminals of one current transformer to the relay and the impedance of the leads from the secondary terminals of the companion current transformer to the relay.

Unequal lead impedances impose different secondary burdens on the two current transformers. The current transformer with the greater secondary burden requires a higher value of core flux and therefore a correspondingly greater exciting current which causes its ratio of transformation to increase beyond that of its companion. Under the extremes of a heavy thru-
fault current and large unbalance in secondary burden the difference in current transformer ratios may become sufficient to produce a value of differential current more than sufficient to cause relay operation. This differential current is actually the difference between the exciting current of the two transformers. In the early use of differential relaying on alternators in the electrical power industry a simple type of relay, with only one current element, was used. The impedances of the secondary burdens were carefully balanced by inserting a compensating resistance between the current transformer having the lower secondary burden, and the relay. This method has been superseded by one which makes use of a relay, which in addition to a differential coil, has one or more restraining coils. The amount of restraining force which opposes the tripping force is a function of the thru current.

In aircraft, neither of these schemes is desirable. In order to keep the secondary wiring as simple as possible the relay preferably should be connected directly across the secondary of one of the current transformers. The relay should be kept as small and simple as possible and restraining elements should preferably be avoided.

a. Steady-State Differential Current

Fig. 55 is an oscillographic record showing traces of fault current, relay current and tripping operations for a thru fault in which the final steady-state current is 760 amperes 400 cycles. This record was obtained under extreme, perhaps very much exaggerated, conditions. These conditions were exaggerated in the following respect.

A resistance of 5.0 ohms was connected between the secondary of one current transformer and the relay, whereas the resistance between the other current transformer and the relay was negligible. The largest resistance be-
Fig. 55 Through-Fault Check on Double C. T. Differential Scheme
Large Unbalanced Secondary Burdens. If = 760 Amperes
between a current transformer and the relay in an actual aircraft would probably be less than 1.0 ohm which corresponds to about 150 feet of AN-18 copper wire or a distance of 75 feet between the alternator and the bus. Fig. 55 shows that the initial (transient) fault current was almost completely offset. This means that the fault current had an initial d-c component almost equal to crest value of the a-c component. The initial relay current which is also the initial differential current contains a relatively large d-c component. It is noteworthy that the relay tripped momentarily after about the first cycle following the application of fault current. However, as both a-c and d-c components of the transient current fell off during the first 0.025 second, following application of fault current, the relay current also fell off and was insufficient to hold the relay in the tripped position. By about this time the voltage regulator started action causing the fault current to increase and after about 0.10 second the differential current was sufficient to cause the relay contacts to chatter until about 0.13 second after which the relay contacts remained closed. This final closing of the relay was caused by the excess of excitation in the current transformer with the 5-ohm resistance between its secondary and the relay, over that of the other current transformer.

1) **Theory for Steady-State Condition**

Fig. 56 shows a schematic diagram of the two-current transformer arrangement. In Fig. 56 the relay R is connected across the secondary of the current transformer designated as C.T. 1 and interconnected to the secondary of C.T. 2. Resistance r represents the resistance of the interconnecting pilot wires. Fig. 56b shows the approximate equivalent circuit of the two-current transformer arrangement of Fig. 56a viewed from the secondary side.
a - Diagram of connections for two current transformer differential protective scheme. (One phase only is shown)

b - Approximate equivalent circuit of two-current transformer differential scheme viewed from secondary.

FIG. 56 - Double current transformer differential circuit
In Fig. 56a

\[ I = \text{primary steady state current} \]
\[ I_1 = \text{exciting current in C.T. 1 viewed from the secondary side} \]
\[ I_2 = \text{exciting current in C.T. 2 viewed from the secondary side} \]
\[ \Delta I = I_2 - I_1 \text{ relay current or differential current} \]

It should be noted that \( I_R \) is the difference between the exciting current in C.T. 2 and in C.T. 1. The factors which determine \( I_R \) are derived in the following:

\[
\frac{I}{N} = I_1 + \Delta I + I_M \tag{59}
\]
\[
E_1 + E_2 = r \, I_M \tag{60}
\]
\[
I_1 + \Delta I = I_2 - I_1 - I_2 \tag{61}
\]
\[
(\frac{Y_{e1} + Y_R}{Y_{e2}}) \, E_1 = \frac{Y_{e1} + Y_R}{Y_{e2}} \, E_2
\]

\[
E_2 = \frac{Y_{e1} + Y_R}{Y_{e2}} \, E_1
\]

\[
\frac{Y_{e1} + Y_{e2} + Y_R}{Y_{e2}} \, E_1 = r \, I_M
\]

\[
E_1 = \frac{Y_{e2} \, I_M \, r}{Y_{e1} + Y_{e2} + Y_R} \tag{62}
\]

\[
I_M \approx \frac{I}{N}
\]

\[
\Delta I \approx \frac{r \, Y_R \, Y_{e2} \, I/N}{Y_{e1} + Y_{e2} + Y_R} \tag{63}
\]

For the oscillogram of Fig. 55, a resistance of five ohms was placed in series with the two current transformers, and the resulting differential current is 0.25 ampere for a fault current of 760 amperes. From the data listed in Table XXIX, the impedance of the relay is approximately 58 ohms and therefore the voltage across the relay is approximately 0.25 x 58 = 14.5 volts. If this voltage is taken into account, both current trans-
Table XXIX
CURRENT TRANSFORMER AND RELAY CHARACTERISTICS

1. Excitation Characteristic of 40/0.333 amp Current Transformer

P 1307085 at 400 cycles.

Note: The current transformer was excited from the secondary winding.

<table>
<thead>
<tr>
<th>Volts</th>
<th>Amp</th>
<th>Admittance</th>
<th>QM</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.36</td>
<td>0.02</td>
<td>0.0085</td>
<td>1,110</td>
</tr>
<tr>
<td>6.9</td>
<td>0.04</td>
<td>0.0058</td>
<td>3,240</td>
</tr>
<tr>
<td>13.9</td>
<td>0.06</td>
<td>0.0042</td>
<td>6,540</td>
</tr>
<tr>
<td>18.0</td>
<td>0.08</td>
<td>0.0045</td>
<td>8,470</td>
</tr>
<tr>
<td>22.2</td>
<td>0.10</td>
<td>0.0045</td>
<td>10,400</td>
</tr>
<tr>
<td>28.0</td>
<td>0.25</td>
<td>0.0089</td>
<td>13,200</td>
</tr>
<tr>
<td>32.0</td>
<td>0.50</td>
<td>0.0156</td>
<td>15,100</td>
</tr>
<tr>
<td>38.1</td>
<td>1.00</td>
<td>0.0263</td>
<td>17,900</td>
</tr>
</tbody>
</table>

2. Relay Characteristics

Name Plate Data:

Differential Protective Relay - Type AVP-208-A

Mfg. Part No. A18B2232

Serial No. LU-126

For use with 200/115 V a-c system only

400-cyle Test:

Volts = 10.5

Amp = 0.18

Impedance = 58.4 ohms

D-C resistance = 7.0 ohms
FIG. 58 - CURRENT TRANSFORMER CHARACTERISTICS

(A) EXCITING CURRENT (RMS) VS VOLTAGE (RMS)

(B) EXCITING CURRENT (RMS) VS FLUX (MAX)
formers C.T. 1 and C.T. 2 operate in the unsaturated region during the final steady-state condition. Then from Fig. 58a the exciting admittance of the current transformers are:

\[ |Y_{e1}| = |Y_{e2}| \approx \frac{0.10 \text{ amp}}{20 \text{ volts}} = 0.005 \text{ mho}. \]

Neglecting core losses and the resistance of the secondary winding, the admittance can be expressed by

\[ Y_{e1} = Y_{e2} \simeq 0 - j 0.005 \text{ mho}. \]

From Eq. (63), the differential current is

\[ \Delta I = \frac{-j 5 \times 0.017 \times 0.005 \times 7.25}{0.027} \]

\[ = - j 0.11 \text{ amp} \]

The final steady differential current shown in Fig. 55 has an rms value of 0.25 ampere which checks the computed value to within 0.11 ampere for a fault current of 760 amperes.

The discrepancy between the measured and calculated value is probably due to C.T. 2 being slightly saturated.

2) Theory for Transient Condition

The approximate equivalent circuit shown in Fig. 56b although valid for transient currents is difficult to apply as \( Y_{e2} \) the exciting admittance of C.T. 2 changes with saturation due to the d-c component in the transient current without an accompanying equal change in \( Y_{e1} \) the exciting admittance of C.T. 1. This difference in saturation is due of course to the inequalities in the secondary burdens of the two current transformers. The oscillogram shown in Fig. 57 is an excellent example of the effect of the d-c component in the fault current. In order to make a differential scheme
of this kind fool proof it is necessary to reckon with fault currents which are completely offset as this type of current imposes the most severe duty on the current transformers.

A rigorous treatment which takes into account the non-linear exciting characteristics, if it were at all possible, would lead to an extremely difficult analysis and to final equations which would probably be so cumbersome as to obscure the essentials. Therefore, in order to simplify the analysis, let us investigate how the flux in the core of the current transformer must build up so that the secondary current is a completely offset wave.

Let \( N \) = number of secondary turns

\[ r + jX = \text{impedance of the secondary burden, including the resistance of the secondary winding but neglecting the secondary leakage reactance} \]

\( i = \text{instantaneous secondary current} \)

\( T = \text{time constant of primary circuit} \)

\( e = \text{induced secondary voltage} \)

\( \phi = \text{core flux in webers} \)

\( I = \text{rms value of a-c component of fault current, considered constant} \)

\[ i = \frac{\sqrt{2} I}{N} \left( \cos \omega t - e^{-t/T} \right) \]

Before proceeding with the derivation it might be well to indicate that we are imposing two conditions which are not satisfied in practice, a fact which however does not invalidate the following analysis. For one thing we are assuming here that the secondary current is the primary current divided by the turns ratio. Secondly we are assuming that the alternating component of the fault current remains constant while actually the a-c component starts out at its transient value and in a few cycles dies down to what would be its
final steady-state value, but by about that time the voltage regulator increases excitation and the fault current increases. In this analysis we are primarily concerned with that time interval during which the d-c component of fault current has appreciable magnitude i.e. before regulator action starts. Taking into account the decrement of the a-c component would unnecessarily complicate the derivation. We may therefore proceed as follows:

\[
\frac{N \, d \, \phi}{dt} = e = ri + \frac{X}{\omega} \frac{di}{dt}
\]  

(64)

\[
\phi = \frac{1}{N} \int_{0}^{t} e \, dt = \frac{1}{N} \int_{0}^{t} \left( ri + \frac{X}{\omega} \frac{di}{dt} \right) dt
\]

\[
= \frac{\sqrt{2} T}{N^2} \left[ \frac{r}{\omega} \sin \omega t + \frac{X}{\omega} \cos \omega t
\right.
\]

\[
- \frac{X}{\omega} e^{-t/T} - rT \left( 1 - e^{-t/T} \right) \]

(65)

If we neglect the reactance of the pilot wire and consider that value of time \( t \) by which the transient component of current has become negligible, we get

\[
\phi = \frac{\sqrt{2} T}{N^2} \left( \frac{r}{\omega} \sin \omega t - rT \right)
\]

(66)

\[
\phi = \phi_{ac} + \phi_{dc}
\]

The a-c component of core flux is

\[
\phi_{ac} = \frac{\sqrt{2}}{N} \frac{r}{\omega} \sin \omega t, \text{ webers}
\]

(67)

and the d-c component of core flux is

\[
\phi_{dc} = \frac{\sqrt{2}}{N} \frac{1}{2} rT, \text{ webers}
\]

(68)

It should be noted that both components of flux vary:

a) directly as the secondary resistance which includes the resistance of the secondary winding
b) directly as the primary current  
c) inversely as the secondary turns squared

In addition the d-c component of flux varies directly as the time constant of the primary circuit.

For example consider a line-to-ground fault similar to that represented by the oscillogram in Fig. 57 except that the fault current wave is completely offset. In order to reproduce a completely offset secondary current the magnetic flux in the current transformer must satisfy the following condition

$$\phi = \sqrt{\frac{2}{N^2}} \left( \frac{I}{\omega} \sin \omega t - rT \right), \text{webers}$$  \hspace{1cm} (69)

The numerical values for Eq. (69) are

$$I = 1520 \text{ amperes from Fig. 57}$$

$$N = 105 \text{ secondary turns}$$

$$\omega = 2 \pi f = 2 \pi \times 400 = 2520 \text{ radians per sec.}$$

$$T = \frac{X_2}{2 \pi f R} \text{ time constant of d-c transient}$$  \hspace{1cm} (70)

From Table XIII

$$X_2 = X_{2G} + X_{2L} \text{ negative sequence reactance}$$

$$X_{2G} = 0.217 \text{ per unit negative sequence reactance of the alternator}$$

$$X_{2L} = 0.005 \text{ per unit negative sequence armature leads to bus}$$

$$R = R_a + R_L$$

$$R_a = 0.024 \text{ per unit d-c resistance of armature at 75^\circ C}$$

$$R_L = 0.019 \text{ per unit d-c resistance of armature leads to bus}$$

Substituting these numerical values in Eq. (70) we get

$$T = \frac{0.222}{2520 \times 0.043} = 0.00205 \text{ second}$$
Current Transformer C.T. 1

Referring to the circuit shown in Fig. 56 and neglecting the impedance of the relay, the secondary burden on C.T. 1, as far as the core is concerned, is the resistance of its secondary winding. As the secondary winding is uniformly distributed around a circular core, the secondary leakage reactance can be neglected.

\[ r = 0.30 \text{ ohm, the resistance of the secondary winding} \]

Substituting all the above numerical values in Eq. (69) we get for the flux in the core of C.T. 1

\[
\phi_1 = \frac{\sqrt{2} \times 1520}{(105)^2} \left( \frac{0.30}{2520} \sin \omega t - 0.30 \times 0.00205 \right)
\]

\[ = (2.32 \sin \omega t - 12.00) \times 10^{-5} \text{ webers} \]  

(71)

It should be noted that the d-c component requires \( \frac{12.00}{2.32} = 5.17 \) times the maximum value of flux required by the a-c component of current. If we express \( \phi_1 \) in kilolines we get

\[ \phi_1 = (2.32 \sin \omega t - 12.0) \text{ kilolines}. \]

Current Transformer C.T. 2

The secondary burden as far as the core of current transformer C.T. 2 is concerned, if the impedance of the relay is neglected, is the resistance of its secondary winding plus the resistance of the pilot wire. If we consider a rather extreme, although not impossible, case i.e. that of the two current transformers 50 feet apart, we have a total length of 100 feet of pilot wire. If this pilot wire is comprised of AN-18 copper as for the conditions under which the oscillograms in Fig. 57 were obtained, its resistance is about 0.64 ohm. Therefore, if the impedance of the relay is neglected, the secondary burden of C.T. 2 as far as its core

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is concerned is that due to the resistance of the pilot wire and the resistance of the secondary winding. Hence \( r = 0.30 + 0.64 = 0.94 \text{ ohm} \).

Substituting the proper numerical values in Eq. (69) we get

\[
\phi_2 = \frac{\sqrt{2} \times 1520}{(105)^2} \left[ \frac{0.94}{2520} \sin \omega t - 0.94 \times 0.00205 \right] \\
= (7.27 \sin \omega t - 37.5) \times 10^{-5} \text{ webers}
\]

Expressed in kilolines

\[
\phi_2 = (7.27 \sin \omega t - 37.50) \text{ kilolines}
\]

Effect of d-c Component and a-c Component in Fault Current

Summarizing we have the equations for the fluxes in the two current transformers as follows:

\[
\phi_1 = (2.32 \sin \omega t - 12.00) \text{ kilolines}
\]

\[
\phi_{1\text{max}} = 14.32 \text{ kilolines}
\]

\[
\phi_2 = (7.27 \sin \omega t - 37.50) \text{ kilolines}
\]

\[
\phi_{2\text{max}} = 44.77 \text{ kilolines}
\]

It must be borne in mind that Eq. (74) and Eq. (75) which are for the fluxes in the respective current transformer cores are derived on the basis that the secondary currents will be completely offset.

The value of 14.32 for \( \phi_{1\text{max}} \), the flux in the core of C.T. 1, is realistic although somewhat greater than would be the case for a completely offset wave of 1520 amp a-c. Fig. 58b shows that under these conditions C.T. 1 has started to saturate and that the exciting current would vary from a minimum instantaneous value of about 0.10 \( \times \sqrt{2} = 0.14 \) ampere (for \( \phi_1 = 2.32 - 12.00 = -9.68 \)) to about a maximum instantaneous value of 0.35 \( \times \sqrt{2} = 0.50 \) ampere. The current scales in Fig. 58 are in rms values.
In the case of C.T. 2, because of its greater secondary burden, we get a correspondingly greater value maximum value of flux, namely \( h_{4.77 \text{ kilolines}} \). Inspection of Fig. 58b discloses that such a value of flux is unrealistic and that the practical matter of the case is that the flux in the core of C.T. 2 fell far short of attaining such a high value. As a result C.T. 2 did not reproduce the fault current in its secondary with the same degree of fidelity as C.T. 1 and the difference shows up as differential current, sufficient to cause relay operation. However, the d-c component in the primary current decays quite rapidly causing the d-c component in the flux of the two current transformers to fall to its residual value which allows the current transformers to recover in a large measure their proper ratio characteristics.

Referring again to Fig. 57, it can be seen that the fault current decreased somewhat immediately after the short circuit was initiated. This is to be expected, as during the initial part of the short circuit the positive-sequence reactance is equal to the transient reactance and starts to increase, eventually approaching the direct-axis synchronous reactance. However, the voltage regulator, which senses the average voltage of the three phases, increases the excitation, increasing the fault current to a final steady value which is actually somewhat greater than the initial value of the a-c component. It is extremely significant that, in the face of this greater final value, the differential current is much less than its initial value. This, of course, is explained by the fact that there is no d-c component in the steady value of fault current and the fluxes in the cores of the two current transformers need be sufficient only to take care of the a-c component of fault current. Hence the steady fluxes are:
\[ \phi_1 \approx 2.32 \sin \omega t \text{ kilolines} \]
\[ \phi_2 \approx 7.27 \sin \omega t \text{ kilolines} \]

The exciting currents taken from Fig. 58b are respectively

\[ i_1 = 0.02 \text{ amp rms} \]
\[ i_2 = 0.07 \text{ amp rms} \]
\[ \Delta i = i_2 - i_1 = 0.07 - 0.02 = 0.05 \text{ amp relay current.} \]

Fig. 57 shows a differential current of \( \Delta i = 0.054 \text{ amp} \).

The computed value and the measured value of relay current are in good agreement. It might be argued that the fault current developed by two alternators has twice the magnitude possible in practice where the current transformers can be subjected to only the current from one alternator. On the other hand, the fault current wave shown in Fig. 57, although somewhat offset, is not nearly completely offset which means that for a completely offset wave of one-half the magnitude a greater d-c component could be expected. In that case the relay would trip.

**Precautions Against Thru Fault Operation**

Several different steps may be taken to render the two-current transformer arrangement of differential protection proof against operation on thru fault. These may be outlined as follows:

a) Equalize the burdens on the secondaries of the current transformers. This can be effected by inserting resistance between the relay and the secondary of the current transformer with the lesser burden. Fig. 59 shows the trace of a fault current of 1520 amp a-c rms when the burdens on the two current transformers was comprised of a 2.5 ohm resistance each. This burden does not include the resistance of the secondary winding. The differential
Fig. 59 Through-Fault Check On Double C. T. Differential Scheme
Excessive Balanced Burdens \( I_p = 1520 \text{Amperes} \)
current shown in Fig. 59 is negligible.

b) Desensitize the relay so that a higher value of internal fault current is required for tripping.

c) Increase the cross-sectional area of the core of the current transformers to reduce the exciting current.

d) Increase the size of the pilot wire to reduce its resistance.

e) Without changing the core and the amount of copper in the secondary winding of the current transformer and in the relay coil, increase the number of turns on both the transformer secondary and the relay coil.

The most practical of these methods is e), increasing the number of secondary turns. Method a) requires the addition of a carefully measured amount of resistance which increases installation and maintenance difficulties which in turn present the hazard of errors. Method b) reduces the effectiveness of the protection against internal faults. Method c) increases the weight of the current transformers unless the circumference is decreased. Method d) increases the weight of pilot wires and is therefore objectionable. Method e) does not decrease sensitivity for internal faults nor does it increase the weight of wiring or of the current transformer but does decrease the danger of operation on thru fault. The disadvantage of increasing the secondary turns arises when the secondary becomes open circuited. Under the abnormal condition, the secondary voltage can rise to high values. Doubling the secondary turns probably does not naturally increase the hazards, but this factor should be checked further.

4) Secondary Burden of Pilot Wire as a Function of Secondary Turns

In order to analyze the effect of increasing the secondary turns in the current transformer and the turns in the relay without increasing the total copper in either, let us consider Eq. (63) which expresses the dif-
ferential current as follows:

\[ \Delta I = \frac{r \frac{Y_R}{N} \frac{I}{Y_{e1} + Y_{e2} + Y_R}}{\frac{n I}{Y_{e1} + Y_{e2} + Y_R}} \]  

(63)

The amount of flux in the relay is proportional to the ampere turns in the relay coil if non-linearity of the iron is neglected.

Suppose that we have an existing system in which

\[ N = \text{the number of secondary turns in the current transformers} \]
\[ n = \text{the number of turns in the relay coil} \]

then if we multiply Eq. (63) by \( n \) we get

\[ n \Delta I = \frac{r \frac{Y_R}{Y_{e1} + Y_{e2} + Y_R}}{\frac{n I}{Y_{e1} + Y_{e2} + Y_R}} \]

(76)

Let

\[ r = R_t + R_p \]

(77)

where:

\[ R_t = \text{resistance of secondary winding of N turns and} \]
\[ R_p = \text{resistance of pilot wire.} \]

For a given pilot wire, \( R_p \) remains constant, regardless of the number of turns in the current transformer secondaries and the relay coils.

Hence

\[ n \Delta I = \frac{(R_t + R_p) \frac{Y_R}{Y_{e1} + Y_{e2} + Y_R}}{\frac{n I}{Y_{e1} + Y_{e2} + Y_R}} \]

(78)

Suppose that we reduce the wire size of the windings and increase the number of turns by a factor \( a \) keeping the winding copper weight the same; then we get

\[ an \Delta I' = \frac{1}{a^2} \frac{r \frac{Y_R}{Y_{e1} + Y_{e2} + Y_R}}{\frac{n I}{Y_{e1} + Y_{e2} + Y_R}} \]

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the ratio of new relay ampere turns to original relay ampere turns is now

\[
\frac{\Delta I'}{\Delta I} = \frac{R_t + \frac{R_p}{a^2}}{R_t + R_p}
\]

The current transformers used in this investigation have 105 turns of #21 AWG copper wire. Fig. 57 shows that with the relay connected directly across the secondary of one such current transformer and 100 feet of AN-18 pilot wire between the relay and the secondary of the other current transformer, a primary thru current of 1520 amp, not completely offset, caused the relay to trip. If these current transformers were used in the same location with the same length of pilot wire, but rewound with 210 secondary turns of #24 AWG copper wire and with the relay coil rewound with twice as many turns of #29 AWG copper as those of #26 AWG copper wire in the existing coil, the effect would be equivalent to having the original 105 turn current transformers connected with 25 feet of AN-18 copper pilot wire instead of 100 feet of the same size of copper pilot wire. The primary current applied to one current transformer necessary to trip the relay would be the same for the 210 secondary-turn current transformers as for the 105 secondary turn current transformers. Thus in going to twice the secondary turns and twice the turns in the relay, a great gain in safety against thru fault operation is achieved without any sacrifice in sensitivity as far as internal fault tripping is concerned. In addition the increased number of turns involves practically no increase in weight. Theoretically another method of safeguarding against tripping on thru fault is to add a compensating resistor between the relay and the secondary of the current trans-
former to which the relay is normally connected directly. This has been done for large alternators in the electric power industry, each installation requiring its own exact value of compensating resistance depending upon the distance between the bus-end current transformer and the neutral-end current transformer. While such an arrangement is feasible for large stationary installations, it is too complicated to be used on aircraft from the standpoint of ease of installation and maintenance.

3. Two-Pilot Wires for All Three Phases

Fig. 60a shows a diagram of connections for a scheme of differential protection for all three phases which uses only one relay and only one pair of pilot wires. This scheme affords practically the same degree of protection for single-line-to-ground faults as the scheme in which there is a relay and pair of pilot wires for each phase. However, it does not protect against line-to-line faults if the ratios and polarities of all the current transformers are alike. On an internal line-to-line fault, the currents in the secondaries of the current transformers in the faulted phases, are 180 degrees out of phase and as a result no current flows in the relay. Fig. 60b shows a modification of the single-relay and two-pilot wire scheme shown in Fig. 60a. In Fig. 60b the current transformers in A and C phases have the same ratio N but they have opposite polarities so that relay current does result on a line-to-line fault between A and C phases. The current transformers in B phase Fig. 60b have the same polarities as the A phase current transformers but have a different ratio namely k N instead of N. With this arrangement relay current results from line-to-line faults between A and B phases and between B and C phases. With such an arrangement different values of sensitivity result depending upon the type of fault and the phases involved. These values are listed in Table XXX and are shown
a - IDENTICAL CURRENT TRANSFORMERS - INEFFECTIVE FOR LINE TO LINE FAULTS.

b - DISSIMILAR CURRENT TRANSFORMERS - DETECT LINE TO LINE FAULTS.

FIG. 60 - DOUBLE CURRENT TRANSFORMERS DIFFERENTIAL SCHEME WITH 2 PILOT WIRES FOR ALL 3 PHASES

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graphically in Fig. 61. Expressions for the relative sensitivities can be determined as follows:

Let \( \Delta I \) = current required to trip the relay

Then \( I_F = N \Delta I \), the fault current on A or C phases on a single-line-to-ground fault to trip the relay (The exciting currents of all the current transformers are neglected).

\[ k I_F = N \Delta I \text{, the fault current for single-line-to-ground fault on B phase to trip the relay} \]

\[ \frac{I_F}{2} = N \Delta I \text{, the fault current for a line-to-line fault between A and C phases to trip the relay} \]

\[ \frac{k}{1+k} I_F = N \Delta I \text{, the fault current for a line-to-line fault between A and B phases} \]

\[ \frac{k}{1+k} I_F = N \Delta I \text{, the fault current for a line-to-line fault between B and C phases}. \]

From Fig. 61 it appears that a practical value for \( k \) is between 0.5 and 0.7. B phase current transformers would have one-half to seven tenths the turn ratio of the current transformers in A and C phases.

a. Advantages

The chief advantage of the two-pilot wire arrangement shown schematically in Fig. 60b is that only one-third as much pilot wiring is required as for the conventional arrangement in which a pair of pilot wires is required for each phase. The advantages are:

(1) Reduced vulnerability because of reduced exposure of pilot wires

(2) Saving in weight

(3) Simpler.

In addition only one relay, as compared with three relays, is required.

b. Two-Pilot Wires, Two Relays for All Three Phases

Where a line has available power at both ends, circuit inter-
<table>
<thead>
<tr>
<th>k</th>
<th>$I_a$</th>
<th>$I_b$</th>
<th>$I_c$</th>
<th>$I_{ab}$</th>
<th>$I_{bc}$</th>
<th>$I_{ca}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.250</td>
<td>1.000</td>
<td>0.250</td>
<td>1.000</td>
<td>0.333</td>
<td>0.200</td>
<td>0.500</td>
</tr>
<tr>
<td>0.333</td>
<td>1.000</td>
<td>0.330</td>
<td>1.000</td>
<td>0.500</td>
<td>0.250</td>
<td>0.500</td>
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<td>1.000</td>
<td>0.500</td>
<td>1.000</td>
<td>1.000</td>
<td>0.333</td>
<td>0.500</td>
</tr>
<tr>
<td>0.750</td>
<td>1.000</td>
<td>0.750</td>
<td>1.000</td>
<td>3.000</td>
<td>0.429</td>
<td>0.500</td>
</tr>
<tr>
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<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>0.500</td>
<td>0.500</td>
<td>0.500</td>
</tr>
<tr>
<td>2.000</td>
<td>1.000</td>
<td>2.000</td>
<td>1.000</td>
<td>2.000</td>
<td>0.667</td>
<td>0.500</td>
</tr>
<tr>
<td>3.000</td>
<td>1.000</td>
<td>3.000</td>
<td>1.000</td>
<td>1.500</td>
<td>0.750</td>
<td>0.500</td>
</tr>
</tbody>
</table>

The above values are the ratio of the amount of current required to trip the relay for the scheme shown in Fig. 60b to that required for the conventional double current transformer with one relay per phase as shown in Fig. 50b.
FIG 61 - COMPARISON OF TRIPPING CURRENTS
rupting devices must be provided at each end of the line. With a single relay it would be necessary to run tripping leads to the opposite end of the line from the relay in addition to the pilot wires. This increases complexity, vulnerability, and weight. The arrangement shown in Fig. 62a uses a relay at each end of the pilot wire and avoids the need for running tripping leads from one end to the other. It requires tripping power to be available at both ends of the line.

For generator differential protection it may also be desired to have a circuit interrupting device at the generator-end of the line to assist in de-energizing the generator by

(a) opening the exciter or alternator field
(b) opening the neutral.

a. **Sensitivity**

From Fig. 62 it is evident that on an internal fault the differential current divides between the two relays. If the impedance of the pilot wire is low compared with that of the relay, then the differential current divides equally between the two relays. Therefore, it takes twice the internal fault current to cause tripping in the two-relay system as it does for the one-relay scheme. In other words the two-relay scheme has one-half the sensitivity of the one-relay scheme.

b. **Thru Faults**

Fig. 62b shows the equivalent circuit of a differential protection scheme using 2 pilot wires and 2 relays for all three phases in which:

\[ Z_W = \text{impedance of total length of pilot wire (going and return)} \]
\[ Z_R = \text{impedance of each relay. The relays are identical.} \]
\[ I = \text{total secondary current on thru fault} \]
FIG. 62 - DIFFERENTIAL SCHEME WITH 2-PILOT WIRES, 2 RELAYS FOR ALL THREE PHASES

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The currents in the circuit are:

\[ I_W = \frac{2E}{Z_W} \]  
\[ I_R = \frac{E}{Z_R} \]  
\[ I = I_W + I_R = E \left[ \frac{2}{Z_W} + \frac{1}{Z_R} \right] = E \left[ \frac{2Z_R + Z_W}{Z_W Z_R} \right] \]

The ratio of the relay current to the secondary current is:

\[ \frac{I_R}{I} = \frac{Z_W}{2Z_R + Z_W} \]  

From Eq. (84) it is evident that, in order to prevent operation on thru fault, the impedance of the relay must be sufficiently great in relation to that of the pilot wires so that the differential current is well below that required to trip the relay. Considerations of economy of weight dictate that the size of the pilot wire be kept as small as possible. Therefore, in order to effect a high ratio of \( Z_R \) to \( Z_W \) the impedance \( Z_R \) of the relay must be made high, if necessary by increasing the number of turns in the operating coil and by increasing the number of secondary turns in the current transformers. Increasing the relay impedance in that manner does not decrease the sensitivity to internal faults.

5. Modified Two-Pilot Wire Scheme

An alternative to the two-pilot wire two-relay scheme described above is one which is shown schematically in Fig. 63a. The secondaries of the thru current transformers on each end feed into the center tap of the primary winding of a transformer T. The secondary winding of transformer T is connected to the relay R. A compensating resistance \( R_W \) equal to one-half
FIG. 63 - MODIFIED 2-PILOT WIRE DIFFERENTIAL SCHEME
the total pilot wire resistance is used as shown. The reactance of the pilot wire is small and can usually be neglected.

The equivalent circuit of this arrangement is shown in Fig. 63b.

a. **Thru Fault**

In the case of a thru fault \( I_I = I_{II} \) and \( I_I \) divides equally in the two halves of the primary winding. This is evident from the following. If the leakage impedance between the two halves of the primary windings (Fig. 63b) are neglected and \( I_I = I_{II} = I \) then because of symmetry:

\[
E_I = E_{II} = E \\
E_I = e_I + (I_I - I_I') \frac{R_w}{2} \\
E = e_I + (I_I - I_I') \frac{R_w}{2} \\
E_I + E_{II} = e_I + e_{II} + I_I' R_w \\
2E = 2e + I_I' R_w \\
I_I' = \frac{I_I}{2}
\]

Note: When \( I_I' = \frac{I_I}{2} \) \( e_I = e_{II} = 0 \)

Under this condition the relay current \( I_R \) is zero as the equal currents in the two halves of the primary winding flow in opposite directions.

b. **Internal Fault**

On an internal fault let \( I_I - I_{II} = I \) be the difference in current. The effect of such an unbalance is similar, as far as relay current is concerned, to the condition that

\[
I_{II} = 0 \\
\text{and} \\
I_I = I
\]

Then we have

\[
E_I = e_I + (I - I_I') \frac{R_w}{2}
\]
\[ E_I = -e_I + I I R_w - 2e_I + I I I R_w \]
\[ 0 = 2(e_I + e_{II}) + I R_w - 2I I I R_w \]
\[ e_I = (I - 2I I I) \frac{R_w}{2} \]
\[ e_{II} = -I I I \frac{R_w}{2} \]
\[ (I - 4I I I) \frac{R_w}{2} + I \frac{R_w}{2} - 2I I I R_w = 0 \]
\[ 4I I I \left( \frac{R_w}{2} + \frac{R_w}{2} \right) = I \left( \frac{R_w}{2} + \frac{R_w}{2} \right) \]
\[ I I I = \frac{I}{4} \]

The differential current in the left hand relay is

\[ I - 2I I I = \frac{I}{2} \]

and that in the right hand relay is

\[ 2I I I = \frac{I}{2} \]

hence the currents in the two relays are the same.

On an internal fault the differential current theoretically divides evenly between the two relays. This modified scheme has been used in underground transmission lines where the resistance of the pilot wire, because of its great length, was quite high.

This scheme however, is not recommended for use on aircraft as it requires the adjustment of the compensating resistance \( \frac{R_w}{2} \) i.e. one-half the total resistance of the two pilot wires.

6. **Direct Acting Trip**

Availability of power for tripping a circuit breaker under fault must be carefully considered in the application of protection.

Separate d-c tripping batteries may be used, but complicate the electrical system and increase vulnerability. Use of a-c voltage from the
protected system may also be considered. Reasonable assurance of tripping power for phase-to-ground faults can be provided by appropriate circuit connections. Assurance of tripping power for phase-to-phase and three-phase faults is more difficult.

Direct acting trip removes the danger of failure of tripping power source and reduces the number of interconnections which must be made. Direct acting trip means that the short-circuit currents themselves act to trip the circuit breaker. Direct acting trip would be particularly advantageous for the scheme shown in Fig. 62 and 63. Investigations should be carried out to determine the feasibility of direct acting trip circuit breakers for differential current protection. This investigation would involve determination of whether reasonable tripping forces could be achieved without use of excessively large current transformers and tripping solenoid while retaining reasonable sensitivity and speed.

7. Current Transformer Design

The present design of the current transformers can probably be improved without seriously increasing their weight, by

a) making the window smaller. This decreases the length of the magnetic circuit

b) increasing the cross sectional area of the core

c) increasing the number of secondary turns.

Each of the above three items would decrease the excitation required by the core. By properly co-ordinating the design of the current transformers with modifications of the operating coil in the relays now in use, nuisance tripping on thru faults can be avoided. However, the sensitivity would be such that the minimum thru fault current to cause trip-
ping would be about twice that required in the scheme using only one relay per pair of pilot wires.

Studies should be made to determine the optimum design of current transformers and differential relays.
V. APPLICATION OF SYSTEM PROTECTION

The arrangement of a power generation and distribution system is greatly influenced by the type of protection that appears best adapted to the needs of the system. Hence, this section will describe the application of various protective methods to the system elements.

Exact evaluation of the relative merits of different circuit arrangements and protective schemes is virtually impossible with available information on failures. It is doubtful that such information will become available and if it were it would rapidly become out dated by changing combat conditions. Hence, comparisons between systems will inevitably be made largely on intangible factors of "judgement". In the discussions of protective schemes in this report comparisons of those factors which can be measured are presented. But to a large degree the results are dependent upon the weighting given to the various factors by "judgement".

A. Alternator Protection

1. Types of Failures

The failures which may occur in an a-c generator can be classified as electrical or mechanical failures as follows;

a. Electrical Failures

1) Armature and Leads to Bus

1') Short Circuits

a') Single line to ground
b') Line to line
c') Double line to ground
d') Three phase
e') Turn to turn
2') Open Circuits
   a') One or more phases open circuited.

2) Field and Regulating Circuit
   1') Loss of Field Excitation
      a') Short circuit in alternator field
      b') Short circuit in exciter field
      c') Open alternator field circuit
      d') Open exciter field circuit

   2') Loss of voltage control

b. **Mechanical Failures**

All the possible kinds of mechanical failure are too numerous to list in detail although they may be classified as follows:

1) Loss of Prime Mover
   1') Engine failure
   2') Constant speed transmission failure
      a') Speed below synchronous speed
      b') Speed above synchronous speed
      c') Blocked rotor

2) Generator Failure - (Blocked Rotor)

2. **Directional Protection**

Since an alternator should not be disconnected from the electrical system in case of external faults, overcurrent protection by itself is inadequate as such protection cannot discriminate between internal and external faults. Such discrimination however is afforded when directional power sensing is used. The problem of directional power protection is receiving detailed study in project 90-1054E under Contract No. AF 33(038)25738 in con-
nection with the development of an Aircraft Directional Circuit Breaker.

Generally directional protection makes use of a wattmeter type relay which has a current coil and a voltage coil. Such a relay can be adjusted to sense real power or reactive power or a combination of real and reactive power. In order for this type of protection to be effective where the load on the bus is entirely passive, (no rotating equipment such as motors on the bus) two or more alternators must be operating in parallel. No reversal of real or reactive power can occur in the case of only one alternator supplying a passive load.

a. Real Power Sensing

One wattmeter element in each phase is so connected as to sense only real power and ignore reactive power. The relay would operate only when the real power flows from the bus into the alternator. The relay could therefore be made to operate on the following electrical failures in the armature and the leads to the bus.

1) Single line to ground
2) Line to line
3) Double line to ground
4) Three phase.

Typical values of currents in faults near the bus on a 2-alternator system are listed in Table XXIV. In addition the relay could operate on the following mechanical failures.

1) Loss of prime power without loss of synchronism
2) Blocked rotor.

Tests made on alternator D, which has an overrunning clutch, indicate a reverse flow of real power of approximately 3.4 kw. when its prime
mover is at standstill. This amount of power is less than 0.10 per unit and a directional protective device which has sufficient sensitivity to trip the alternator for this condition could remove alternators already on the bus when another alternator is synchronized out of phase with the system. Fig. 42 shows that with three alternators on the system, the reverse real power in them can be well above 1.0 per unit when a fourth alternator is connected out of phase to the system (broken ellipse for n=4). With only one alternator on the bus and a second alternator is synchronized out of phase the reverse real power in the alternator on the bus may be greater than 3.0 per unit.

The results of blocked rotor tests are listed in Tables XXV and XXVI. The values of real power listed in those tables indicate that the sensitivity required of a real power sensing directional device is such that alternators already on the system might be tripped off when another alternator is synchronized out of phase to the system. Such nuisance operations can however be prevented by means of adequate time delay.

b. Reactive Power Sensing

If the voltage to the voltage coil, of the wattmeter type relay discussed above, is retarded 90 degrees the relay becomes sensitive to reactive and ignores real power. The same types of electrical failures can be cleared with reactive power sensing as with real power sensing. In addition, reactive sensing will cause relay operation in the case of:

1) Complete or partial loss of field excitation.

Complete loss of field excitation could be caused by an open circuit in the alternator field circuit. In the case of an open circuit in the exciter field circuit, the alternator would receive some excitation because of the
residual magnetism in the exciter. Partial loss of field excitation could be caused by

   a) Faulty voltage regulating circuit

   b) Partial short circuit in the alternator field winding.

Reactive power sensing would also cause relay operation in the case of blocked rotor, but would not cause operation in the case of prime mover failure without loss of synchronism.

Eq. (49) and Eq. (50) Section III-C-4 show that in the case of complete loss of field excitation the reverse reactive power lies somewhere between a value of 0.54 and 0.89 per unit. A reactive power sensing directional device could be made sufficiently sensitive to remove the alternator from the system with little likelihood of nuisance operation during synchronizing on a system operating with more than one alternators on the bus.

In general greater sensitivity might be expected for reactive power sensing than for real power sensing except in the case of faults with appreciable resistance.

There might be some advantage in a directional protective device which is sensitive to some combination of real and reactive power. This can be achieved by retarding the voltage to the voltage element of a normally real power sensing device thru an angle smaller than 90 degrees.

C. Disadvantages of Directional Protection

The speed and sensitivity of directional protective schemes is limited by precautions against nuisance tripping. To prevent nuisance tripping the relay is made insensitive to values of reverse power below a certain limit. In addition a time delay can be incorporated to prevent tripping on normal short time reversals of power flow. Reverse power flow can take

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place where there is no failure under the following conditions:

1) Synchronizing

If an alternator is switched to a live bus when its voltage is appreciably out of phase with the bus voltage, swings of real and reactive power result (Fig. 43). If the relays are too sensitive the incoming alternator and the alternators already connected to the bus would be tripped off. This is particularly true if real power sensing is used. If reactive power sensing is used, the incoming alternator would be tripped off under some conditions of synchronizing, but there would be little likelihood of tripping alternators already on the bus as shown in Fig. 42. Removing the incoming alternator from the system when synchronized far out of phase is probably desirable.

2) Sudden Change in Prime Mover Speed

In the case of two or more paralleled alternators driven from the main engines through constant speed drives, if the speed of one engine is suddenly changed, the division of real power between the alternators is also changed for an appreciable time period because the governor response of the constant speed drive is fairly slow. Under light load conditions such a change in speed could produce reversal of real power flow and if the directional relay senses real power it would operate. If reactive power is sensed relay operation would be unlikely for this condition. Relay operation under this condition would be undesirable and would be considered a nuisance operation.

3) Loss of Speed Sense

If the indication to the governor is improperly one of a loss of speed or large deficiency in speed the governor will increase the amount of
prime power to that alternator. As a result this alternator will generate more than its share of the real power and under light load conditions the real power in the healthy units may reverse. If real power sensing is used the directional relays would cause the healthy units to be disconnected from the system. Relay operation under these conditions is undesirable. The same situation would result from one alternator being driven above synchronous speed for any reason whatsoever.

4) Loss of Voltage Sense

If the voltage sense in one of several alternators operating in parallel is lost, the excitation of that alternator will go to ceiling value. Under light load conditions this can cause reactive power to flow into the healthy alternators and if the directional relays use reactive power sensing the relays on the healthy alternators would operate and disconnect them from the system. Under this condition the reverse reactive power in a healthy alternator may be as much as $1.4$ per unit as shown in Fig. 25.

d. Application of Directional Protection

Directional protection will take care of faults not covered by differential relays such as loss of field excitation and blocked rotor. Such faults do not give rise to differential current. In addition to such coverage directional circuit breakers can afford back-up protection to differential relays on short circuits except for low impedance three phase faults. However, since it is necessary to sacrifice sensitivity and speed in order to prevent nuisance operations the directional protective schemes are less sensitive and slower than differential schemes and are not suitable for replacing differential protective schemes.
3. Generator Differential Current Protection

Methods of differential protection have been discussed in Section IV-D. Those methods that appear best adapted to generator circuit protection are:

1. One current transformer and relay per phase (Fig. 50a)
2. Two current transformers and one relay per phase (Fig. 56)
3. Double current transformer method with pilot wires for all three phases (Fig. 60).

The former method is applicable only where the distance to the main bus is short or where it is used in combination with multiple channels of overcurrent protection.

The latter method using only two pilot wires appears to have considerable advantage. This arrangement applied to the generator circuit is shown in Fig. 64. This method would be particularly effective if the following design features can be achieved.

1. Neutral current transformers housed in generator terminal box.
2. Current transformers at the main bus circuit breaker an integral part of the circuit breaker.
3. Direct acting trip.

Building the current transformers into their respective associated equipment should avoid difficulty with incorrect polarity and turns ratio. The number of leads which must be brought out of the equipment is also thereby reduced.

4. Comparison of Two Types of Generator Circuit Protection

There has been considerable discussion regarding the choice between the two possible arrangements of generator circuit protection where the main bus is located at some distance from the generator. One arrangement shown in
FIG. 64 - ALTERNATOR CONTROLS AND PROTECTIVE
Fig. 1 uses only differential protection. The other arrangement shown in Fig. 2 uses a combination of differential current and overcurrent protection. The obvious advantage of the circuit shown in Fig. 2 is the provision of a multiple number of individually protected channels for the generator current. Hence, in the event of a fault on one of these cables the overcurrent protectors of the faulted cable would remove the cable from the circuit without removing the generator from the system. Thus the generator would continue to deliver its normal share of power to the system. For the differential protection shown in Fig. 1, a fault on any power cable would cause the generator to be removed from the system. This generator would be unavailable for further service until the fault had been cleared by maintenance.

However, several other factors should be considered in comparing these two arrangements. This section of the report will be devoted to a discussion of the relative merits of these two methods of generator circuit protection. All comparisons will be based on the assumption that the overcurrent devices indicated are current limiting fuses as described in specification MIL-F-5372 dated 23 October, 1950.

a. Generator Circuit Description

1) Differential Protection

All physical comparisons between the two protection arrangements will be based on a 40-KVA alternator mounted at the engine and driven by a constant speed drive. The distance from the alternator to the winding or nacelle bus will be assumed to be 10 feet and the distance from this bus to the main bus in the fuselage will be assumed to be 50 feet. The arrangement shown in Fig. 64 will be used for the comparison. Differential protection is provided throughout the full length of the generator leads by three current transformers mounted close to the generator circuit breaker and three
current transformers mounted internally on the alternator. Mounting the current transformers internally on the alternator permits only one common neutral lead to be brought out of the machine. Since this lead needs only to be connected to ground, at some point on the engine it can be made very short. This type of connection permits a material saving in weight. The current transformers at each end of the line are connected in parallel with polarities arranged to detect all types of faults. A complete description of the physical arrangement and sensitivity of this type of differential protection is covered in Section IV-D. This type of differential scheme requires only two control leads. The voltage regulator, exciter control relay, circuit breaker, and other control devices shown in Fig. 61 are standard controls connected in the circuit in the usual manner.

The generator circuit shown in Fig. 61 can be modified so that multiple circuits are used for the greater portion of the transmission line. No fuses are used in these multiple channels or any other part of the power circuit. The advantage of this arrangement is that it permits a considerable saving in the weight of the power cables over the single cable per phase.

2) Differential and Overcurrent Protection

Fig. 2 shows the arrangement of controls and the power circuits used in this type of protection scheme. In this arrangement differential protection is provided for the generator leads which run from the alternator to the wing or nacelle bus. The differential current relay is mounted close to the nacelle bus. Since this relay uses only three current transformers the neutral lead of each phase must be brought out to the relay. Thus with this type of differential protection, six power leads must be brought out from the alternator.

The power circuit from the nacelle bus to the fuselage bus is a
multiple channel arrangement with four wires per phase. Each wire is provided with overcurrent protection at both ends. These cables are selected such that if one circuit becomes open the remaining intact circuits will be capable of delivering normal power to the system. This type of network requires a fairly extensive bus arrangement at both ends of the circuit.

b. Factors of Comparison

To obtain a complete picture of the advantages of one system over the other the generator circuits shown in Fig. 64, Fig. 2, and the multiple cable modification of the system shown in Fig. 64 will be compared with respect to weight, main bus exposure, effect of faults on both the power and control systems, relative speed of protection, and maintenance and operation of the system. The systems shown in Fig. 64, Fig. 2, and the modified system of Fig. 64 will be designated as Systems I, II, and III respectively.

1) Weight

The weights of the three systems are compared in Table XXXI. The table does not show total weights, but indicates quantitatively only the net saving in pounds of Systems I and III over System II. The table shows that a material weight saving can be made using differential protection in conjunction with a multiple number of cables. Even the single cable per phase shows weight saving over System II. The cables were selected on the basis that the generator was a 40-KVA machine mounted in the aircraft as previously mentioned. It was assumed in each system that the power cables were bundled in groups of three over the greatest part of the circuit to maintain exposure area to a minimum. The cable sizes were determined from the paper

WADC TR 52-49
<table>
<thead>
<tr>
<th>Item</th>
<th>Generator System Weights in Lbs</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Multiple Channel Leads</td>
<td>None</td>
</tr>
<tr>
<td>2. Single Channel Leads</td>
<td>49.7</td>
</tr>
<tr>
<td>3. Neutral Leads</td>
<td>Negligible</td>
</tr>
<tr>
<td>4. Fuses and Fuseblocks</td>
<td>None</td>
</tr>
<tr>
<td>5. Buses Other Than Main Bus</td>
<td>None</td>
</tr>
<tr>
<td>6. Differential Protection Leads</td>
<td>1.4</td>
</tr>
<tr>
<td>7. Weight of all other items in</td>
<td>X</td>
</tr>
<tr>
<td>Generator Circuit</td>
<td>Total Weight</td>
</tr>
<tr>
<td>Net Saving over System No. II</td>
<td>6.2</td>
</tr>
</tbody>
</table>
by Schach and Kidwell* which bases the cable current rating on the number of cables in the bundle. The weights of the differential control leads are considered in Table XXXI and it is assumed that these leads would be bundled with the power leads to maintain exposure to a minimum.

2) **Main Bus Exposure**

The comparable part of the main bus exposure of the two methods of protection is considered to be the areas shown in Fig. 65 and Fig. 66. A fault or sufficient damage in this area would cause the generator to be lost and also any distribution buses supplied only from the main bus. Therefore, it is highly desirable to keep the exposure area of this bus to a minimum. The areas of main bus exposure of the three systems are shown in Table XXXII under item No. 1. As can be seen from the table the main bus exposure of the differential protection scheme is only 65% of the bus exposure area of the overcurrent protection scheme. In both types of generator circuits the main bus should be provided with its own differential protection circuit to eliminate the possibility of a prolonged fault on the bus. Such a fault might very well cause the system to pull apart at several points. This would be especially true of a system which employs overcurrent protection extensively. (See section on main bus and tie circuit protection, V-B).

3) **Nacelle Bus Exposure**

System II is the only one of the three systems which has a nacelle bus and its subsequent exposure problems. A typical arrangement of this bus is shown in Fig. 67. The approximate value of this area is listed in Table XXXII. A fault on this bus could have very serious effects on the rest of

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*"Continuous Current of Bundled Cable for Aircraft" by M. Schach and R. E. Kidwell, Jr. Presented at AIEE Conference on Air Transportation. Los Angeles, California, October 1951.
## Table XXXII

COMPARISON OF GENERATOR CIRCUIT EXPOSURE AREAS

<table>
<thead>
<tr>
<th>Location</th>
<th>Exposure Area Sq. Ft.</th>
<th>Result of Fault in this area</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>System I</td>
<td>System II</td>
</tr>
<tr>
<td>1. Main bus</td>
<td>.93</td>
<td>1.42</td>
</tr>
<tr>
<td>2. Nacelle bus</td>
<td>0</td>
<td>1.11</td>
</tr>
<tr>
<td>3. Differentially Protected Power Leads</td>
<td>4.46</td>
<td>1.49</td>
</tr>
<tr>
<td>4. Differential Protection Control Leads</td>
<td>1.15</td>
<td>.92</td>
</tr>
<tr>
<td>5. Total Exposure to Faults</td>
<td>6.52</td>
<td>11.7</td>
</tr>
</tbody>
</table>
the system. Such a fault would be supplied from both the generator and the system. If the impedance of the fault permitted excitation to reach ceiling, the exciter protection relay would open the field and generator circuit breaker. However, the minimum time in which this relay could operate would be several seconds. This is much too long to allow a fault of this nature to remain on the system. This amount of time would be long enough to permit a fuse protected system to pull apart at several possible points. (A more complete explanation of this phenomena is given in a separate Section V-E-1). If the impedance of a fault on the nacelle bus is large enough to prevent the exciter from going to its ceiling voltage the fault may persist. Under these conditions the generator could be damaged beyond repair and the rest of the system would probably break down in the manner previously mentioned. Therefore, elimination of this bus exposure is obviously one of the big advantages of the differential protection scheme over the overcurrent system.

4) Effect of a Fault on Generator Power Leads

If a fault occurs on any of the power leads of a generator circuit which employs complete differential protection (Systems I and III), the circuit breaker will be immediately tripped and the generator will be removed from the system. If the generator circuit uses a combination of differential and overcurrent protection a fault which occurs in that part of the circuit which has differential protection will cause the generator to be removed from the system. A fault which happens in that part of the circuit which is protected by fuses will cause only the faulted cable to be removed from the generator circuit. The generator will continue to supply power to the rest of the system. This is the outstanding advantage which the com-
bined differential and overcurrent scheme has over the generator circuit which use complete differential protection. The exposed areas (Protected area) which are differentially protected are listed under item 3 in Table XXXII for the three systems.

5) Effect of a Fault on Differential Control Leads

Systems I and III which employ only differential protection have a control circuit similar to that shown in Fig. 68a. In this scheme only one relay is required which is connected in parallel with the control leads as shown in the figure. If one of the control leads becomes faulted to ground (Case 1) the control circuit will continue to function properly. If one of the control leads accidentally opens, (Case 2) the differential relay will be energized and consequently remove the generator from the system. In the event that the two control leads become shorted to each other (Case 3) the differential relay will be shorted and the generator circuit will be without any protection. The most common fault in this circuit would probably cause the loss of the generator rather than the loss of only the protection.

System II which uses differential protection over only a portion of the power circuit, has a control circuit similar to that shown in Fig. 68b. In this circuit each current transformer operates its own relay. The contacts of each relay are connected in parallel. Therefore, only two control leads are required to run from the differential relay to the circuit breaker. If one of these leads is faulted to ground (Case 1), the system loses its protection. If an open circuit occurs in one of the control leads (Case 2), the system will also lose protection. In the event that the two control leads become short circuited together (Case 3) the circuit breaker will be
FIG 68 - DIFFERENTIAL CURRENT PROTECTION CIRCUITS
tripped and the generator will be removed from the system. It is probable that a fault on this control circuit, is more likely to cause loss of protection rather than the loss of the generator. The relative disadvantages of these failures are discussed in Section IV-D.

The exposure area of the control leads is listed under item four in Table XXXII.

6) **Total Exposed Area Subject to Faults**

Item 5 in Table XXXII gives the total fault exposure area of each system. This area is the total projected area of the power cables, differential control cables, and bus exposure areas from the alternator terminals to the main bus. From the table it can be seen that Systems I and III have considerably less exposure than System II. This is another distinct advantage of the differential scheme over the overcurrent protection system since any energized conductor no matter how well protected represents some hazard.

7) **Maintenance and Operation**

The combination differential and overcurrent protected system is considerably more complex than the generator circuit which use only differential protection. This factor is apparent when Fig. 64 is visually compared to Fig. 2. In order to ascertain that the circuit in Fig. 2 will function properly the fuses in the primary system should be checked after every flight. It is doubtful if such maintenance checks would be followed in practice. Therefore, it is very likely that many aircraft which are not consistently checked would operate with the protection scheme in a dangerous condition such that should a fault occur the major portion of the primary system would be lost.
Systems I and III which use complete differential protection require practically no maintenance in the power or differential control circuits. This is true because as soon as a fault occurs on the power or control leads the generator is removed from the system and it is readily apparent that maintenance is required. There is only one type of fault on the control leads (as explained previously) where the circuit would lose protection. It would be a very rare case when a fault of this type would occur.

8) **Relative Speed of Protection**

The relative speed of operation of fuses and differential protection is apparent from Fig. 69. The melting characteristic of a typical fuse of 1/3 alternator rating is obtained from Fig. 48 by re-plotting the curve using the alternator rating instead of the nominal fuse rating as the base current. A fuse of 1/3 alternator rating was used, because for a fuse arrangement using four cables per phase (Fig. 2), three of the cables must be able to continuously carry rated alternator current.

Also shown in Fig. 69 is the combined current-time characteristic for a differential relay and circuit breaker. A differential current of approximately 0.25 per unit will cause the relay to close. The operating time for the differential relay and circuit breaker as obtained from test and as specified (MIL-R-5296, Exhibit MCREXE 21-58) is from 7 milliseconds to 14 milliseconds respectively.

Fig. 69 shows that differential protection is faster than fuses unless 11.0 per unit current is encountered. In Section III-D, the maximum short-circuit current from a single alternator is about 7.0 per unit. Therefore, the protection afforded by differential protection is usually faster.
1.0 - MELTING CHARACTERISTIC OF A TYPICAL FUSE OF 1/3 ALTERNATOR RATING

0.1 - SPEC. LIMIT

0.01 - MINIMUM ON TEST

0.001 - OPERATING TIME OF DIFFERENTIAL RELAY AND CIRCUIT BREAKER

TIME - SECONDS

CURRENT - PER UNIT ALTERNATOR RATING

FIG 69 - RELATIVE SPEED OF PROTECTION

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than that afforded by fuses.

Fig. 69 also shows that since differential protection detects
differential currents of 0.25 per unit, it is more sensitive than fuses.

c. Summary

The advantage of the combination differential and overcurrent
protection scheme is:

(1) A short circuit on the overcurrent protected feeders will not
result in loss of the generator.

In this method, failure of the differential protection is not likely to
trip the generator.

The advantages of the differentially protected scheme are:

(1) Less weight
(2) Less main bus exposure
(3) No nacelle bus exposure
(4) Less total exposure
(5) Simpler for ease of understanding and maintenance
(6) Faster action for most faults
(7) No co-ordination problems with protection in other circuits.

In this method, failure of the differential protection is likely to trip
the generator.

It is judged that the advantages of differentially protected
schemes of System I or III outweigh those of the combination differential
and overcurrent scheme of System II.

The advantages of the differentially protected system as compared
with the combination overcurrent and differentially protected system de-
creases as the distance from the generator to the main bus is shortened.
However, in the interest of standardization, the differentially protected system should be adapted for all a-c aircraft power systems.

B. Main Bus, Feeder, and Tie Circuit Protection

1. Main Buses

The main bus system with its associated tie circuits is the backbone of the power supply system. Arrangements of this part of the power system should consider the following factors:

(1) Minimum bus exposure
(2) Simplicity of operation
(3) Alternate sources of supply for essential loads
(4) Relationship to source and distribution system protection problems.

The vulnerability of a main bus is undoubtedly related to the area or volume which it occupies. Hence, electrical circuit arrangements that permit the most compact physical bus structure are highly desirable.

In military aircraft the conditions most conducive to main bus failure are those occurring in combat. Under the stress of battle successful manual operation of the electrical system can hardly be expected. Therefore, any protective or throwover schemes for the electrical system should be capable of automatic operation.

Military aircraft must also be capable of operating with a minimum amount of maintenance. Since an armed forces activity must be expanded rapidly in time of need, it is very likely that the training of maintenance personnel will always be less than would normally be desirable. Hence, there is considerable advantage in using the simplest possible systems consistent with weight and reliability requirements.

It is considered necessary that essential loads be supplied from
more than one source. However, more than two sources of supply would seldom be justifiable from the standpoint of reliability. In fact more than two paths may be more disadvantageous because of the increased complexity with a relatively small improvement in useable reliability. Short circuits in a well designed system should be a rare event even under combat operating conditions.

The incoming source and outgoing feeder circuits are connected to the main bus. In the event of a short circuit on the main bus all of these related circuits may be lost. Therefore, it is important that the maximum possible reliability be obtained for the main bus by careful installation techniques. But even with the best possible methods a main bus fault may still occur and adequate bus protection is needed to rapidly divorce the faulted bus from the rest of the system. The main bus system and protection must be closely co-ordinated with the arrangement and protection methods used by the source and load circuits. This is particularly true where overcurrent protective devices are used, as these devices are responsive to short circuits in other parts of the system. Main bus systems which use a protection scheme that permits the simplest possible arrangement of feeder, source, and load circuits have many obvious advantages.

The main bus is a vital part of the power supply system due to the many interconnections at that point. Since a short circuit on the main bus means the loss of connected circuits, it would appear that maximum reliability would be obtained by having one main bus for each generating source. However, for a large number of sources such an arrangement would have the following disadvantages compared to a system using a fewer number of buses.
(1) More physical and electrical complexity
(2) Greater weight due to interconnecting circuits
(3) Larger exposed area of the source system.

The selection of the number of main buses is largely a judgement evaluation. For non-combat operation there is little doubt that a system with two main buses possibly at a single location would be the preferable arrangement. For combat operation there is insufficient factual data on which to base a decision. However, experience with malfunction of electrical systems in non-combat operation tends to favor the simplest possible system. It is believed that far more failures will result from improper operation and inadequate maintenance of complex systems than will result from other causes on a simple double source system with fast trouble-free protective schemes. Therefore, serious consideration should be given to the adoption of systems utilizing two main buses at two different physical locations.

a. Main Bus Protection Methods

Methods of protecting the main bus include the following types:

1) Overcurrent
2) Ground current
3) Differential current.

The advantages and disadvantages of these types of main bus protection will be discussed briefly in the following paragraphs.

1) Overcurrent Protection

Some of the fuse-mesh networks that have been used depend on overcurrent protection in the source and feeder circuits to provide bus protection. This form of protection has the following disadvantages:
(1) Poor Selectivity
(2) Slow Protection.

To illustrate the selectivity problem the system shown in Fig. 70 will be used. This system shows a two-generator system using multiple channel overcurrent protection devices in the generator, feeder, and tie circuits. Although this circuit shows only a two-generator system with direct tie circuits, it can be shown that any multiple-generator system with any interconnected network between the buses can be resolved to the same simple arrangement. Hence, the comments which follow will apply in general to systems with overcurrent protection in the circuits connected to the main buses.

For this system it will be necessary to determine the relationships between fuse ratings for generator circuit fuses A and B and the tie line fuses. This relationship should indicate whether the fuses provide both bus and line protection. To study these rating relationships it will be helpful to assume short circuits at several line and bus locations as shown in the Fig. 70. Fuse ratings and circuit ratings will be used interchangeably in this discussion, although they may actually differ due to different rating methods for the fuses, generators, and cables. Three channels are used in the example in Fig. 70, although four channels are preferable to give better selectivity.

a) Line Protection

Fault 1 in Fig. 70 requires that the $A_1$ and $B_1$ fuses in the faulted line blow earlier than the $T_1$ fuses. That is, one $B_1$ fuse must be faster than three $T_1$, two $A_2$, or two $B_2$ fuses. This should be no problem. Even if there were, trouble could be avoided by the use of more than three...
FIG. 70 - TYPICAL 2-ALTERNATOR SYSTEM WITH MULTIPLE CHANNEL OVERCURRENT PROTECTED TIE LINE.
channels in the generator circuit.

Similarly with fault 2 it is required that one $T_1$ fuse operate faster than two $T_1$, three $A_1$, or three $B_1$ fuses. This likewise should be no problem. Hence, for the generator system shown there should be no serious line protection problem.

b) Bus Protection

Consider first fault 3 on main bus 1. For this fault the three $A_1$ and three $T_1$ fuses must trip, but the three $A_2$ and three $B_2$ fuses must not trip. Therefore, the $T_1$ fuses must be faster than the $A_2$ or $A_1$ fuses. If fuses with the same current-time characteristics are used for $A_1$ and $T_1$, then the $T_1$ fuses will have to be of a lower rating probably by a factor of 2 or more for safety. This may be an adequate tie line transfer rating but it is a factor that would have to be carefully checked. Thus main bus protection requires that the tie line fuses be faster than the generator circuit fuses.

Next consider a short circuit on junction bus 1 - fault 4. It is important to note that if no other form of bus protection is used this fault would be detected and removed from the rest of the system only if generator 2 is in operation. If generator 1 is the only generator on the system at the time the fault occurs, power to the whole system would be lost and the short circuit would persist until the exciter protection relay opened the circuit breaker. Depending on the type of fault it may be a relatively long period of time before the exciter protection relay operates. This is obviously a hazardous condition.

For fault 4 it would be desirable that the three $B_1$ fuses trip before the three $A_1$ fuses for clarity to maintenance personnel. However, this
is not essential. It is essential that the three $B_1$ or three $A_1$ fuses trip before the three $T_1$ fuses. This means that the $T_1$ fuses should have a slower trip characteristic or higher rating than the $A_1$ fuses. But protection of the main bus required exactly the opposite relationship between the tie line and generator circuit fuses. Thus, faults on the junction bus and main bus present conflicting requirements. Whichever relationship is adopted as the best compromise improper co-ordination will be obtained for the other fault condition.

Even more serious co-ordination problems are evident where there is only one other generator connected to the line at the time of short circuit on the nacelle bus - fault 4. Since $A_1$, $B_1$, $A_2$, and $B_2$ fuses all have the same rating and carry the same fault current, it is possible for $A_2$ or $B_2$ fuses to open before $A_1$ or $B_1$ and hence, divorce that phase of the good machine from the system.

This discussion shows that proper co-ordination cannot be obtained with overcurrent devices in both the generator and tie line circuits for bus faults. Therefore, other forms of fast protection should be considered. Differential protection is one possible type that could be considered for either the generator or tie line circuits. In either case these circuits though differentially protected will not provide bus protection. Thus forms of bus protection must be considered. Bus protection can probably be very fast. Hence, overcurrent line protection in combination with fast bus protection would still be a feasible arrangement.

2) **Ground Current**

Ground current protection of the main bus can be obtained by in-
sulating the housing which incloses the bus from ground except for one connection through a current relay. In the event of current flow through the relay a ground fault on the bus is indicated and the relay should trip all sources of current from the bus. However, ground protection would be sensitive only to ground faults and thus may not be widely applicable to three phase systems.

3) Differential Current

Several types of differential current protection schemes have been described in Section IV. Differential protection can be effectively applied to bus protection as will be illustrated by the description of the following four methods.

a) One Relay Per Phase Per Bus, Fig. 71a

In this method the current transformers of each phase are interconnected to a current relay. This is the most straightforward method and since the circuit breakers are physically close together there is little disadvantage in the six leads between current transformers as is true for generator or line differential protection. Three differential relays are required and a reliable source of tripping power must be assured. Still further reliability can be assured for the relays by using a fourth one in a neutral connection. This assures that all faults will be detected by two relays and thus provides greater assurance of tripping the circuit breakers.

b) One Relay Per Bus, Fig. 71b

This method of differential bus protection employs one current transformer with a different turns ratio and one current transformer with polarity reversed. This method of connecting the transformers has been
CIRCUIT BREAKERS TO TRIP COILS ON EACH CIRCUIT BREAKER.

FIG. 71a- DIFFERENTIAL BUS PROTECTION (ONE RELAY PER PHASE PER BUS)

CIRCUIT BREAKERS TO TRIP COILS ON EACH CIRCUIT BREAKER.

FIG. 71b- DIFFERENTIAL BUS PROTECTION (ONE RELAY PER BUS).
previously discussed. Its advantages are the elimination of two relays and in the simplification of the interconnections. Reliable tripping power must still be assured for this method of protection.

c) **Direct Acting Trip - Three Trip Coils per Circuit Breaker, Fig. 72**

The problem of providing a reliable source of tripping power gives emphasis to the desirability of a direct acting trip on the circuit breaker. This might be accomplished by some form of differential relay with a mechanical linkage to each circuit breaker; Fig. 72 shows an arrangement with a separate differential trip for each circuit breaker. This method is not as sensitive as the arrangements previously discussed. But since bus faults are likely to involve substantial currents, some reduction in sensitivity may not be serious. The number of circuit breakers that could be connected to each main bus would necessarily be limited to that for which adequate sensitivity could be attained.

d) **Direct Acting Trip - One Trip Coil Per Circuit Breaker, Fig. 73**

This arrangement may simplify the tripping mechanism on the circuit breaker and hence may possess some advantage over that shown in Fig. 72. As previously mentioned, it is not yet known whether direct acting trip for differential protection is feasible.

2. **Tie Circuits**

The discussion which has already been presented on generator and bus protection indicates that the following methods are desirable forms of protection.

(a) **Generator - Differential and directional current**

(b) **Bus - Differential**

It is desirable to have double channel feeder service to load
CIRCUIT BREAKERS

CURRENT TRANSFORMERS

TRIP COILS ON CIRCUIT BREAKERS

ALTERNATOR, TIE LINE, & FEEDER CIRCUITS.

FIG. 72: DIFFERENTIAL BUS PROTECTION—DIRECT ACTING TRIP (THREE TRIP COILS PER CIRCUIT BREAKER)

FIG. 73: DIFFERENTIAL BUS PROTECTION—DIRECT ACTING TRIP (ONE TRIP COIL PER CIRCUIT BREAKER)
buses provided from different main buses. Feeder protection is simplified if transfer currents between main buses do not need to occur through the feeder circuits. Hence, a reliable tie system between main buses is considered essential. The arrangement and protection of the tie system will depend greatly on the number of main buses used. Some typical arrangements will be discussed.

a. **Two Main Buses**

With two main buses, the buses should be interconnected with two or more tie circuits. These tie circuits can be protected by either overcurrent or line differential current protection. The latter arrangement may be the arrangement using a single trip coil on each circuit breaker as shown in Fig. 74. For overcurrent protection (Fig. 75) the circuit protective element preferably should be capable of being tripped by bus differential protection, as this would be faster than the overcurrent protection in the tie circuits and the fault would be more definitely located for the maintenance personnel. With overcurrent protection in the tie circuits the number of lines should be not less than three and preferably four per phase to assure selectivity. If the feeder circuits uses overcurrent protective devices, the co-ordination between these circuits and the tie circuits must be checked.

b. **Four Main Buses**

Although there appears to be a definite advantage in the simplicity of two main buses a greater number than two may sometimes be considered desirable. A greater number of buses also reduces the number of circuits per bus which may help in simplifying bus differential current protection.
FIG. 74 - OVERLAPPING DIFFERENTIAL PROTECTION FOR ALTERNATOR, BUS, AND TIE LINE CIRCUITS.

FIG. 75 - COMBINATION DIFFERENTIAL AND OVERCURRENT PROTECTION FOR ALTERNATOR, BUS, AND TIE LINE CIRCUITS.
Four buses may be interconnected in a number of arrangements. Three common arrangements are:

(1) Straight bus (Fig. 76)
(2) Ring bus (Fig. 77)
(3) Synchronizing bus (Fig. 78 and Fig. 79).

For systems such as these the co-ordination problems with over-current protection on the tie circuits become very difficult and may often result in insensitive or slow protection. Therefore, for these more complex arrangements overlapping differential protection is considered desirable.

A four-main bus arrangement which offers advantages for a system having four or more generators is shown in Fig. 80. In this arrangement four main buses are used but they are at only two physical locations. Thus this arrangement offers some of the added reliability of four buses and also retains the simplicity of the two-bus arrangement. Either differential or multiple channel overcurrent protected tie circuits may be used with this circuit.

3. Feeder Circuits

The feeder circuits are those circuits which feed power from the main power buses to individual load buses throughout the airplane. For load buses not supplying equipment essential to flight single channel feeder service may be adequate. However, on military aircraft most load buses require multiple channel service to assure continuous power. The number of channels should be limited to two for simplicity of operation and maintenance unless a greater number are required to permit the use of a particular protection scheme. In the following discussion several methods and arrangements of
FIG. 76 - STRAIGHT BUS ARRANGEMENT.

FIG. 77 - RING BUS ARRANGEMENT.

FIG. 78 - SYNCHRONIZING BUS ARRANGEMENT.
FIG. 79 - ALTERNATE SYNCHRONIZING BUS ARRANGEMENT. (TO REDUCE EXPOSURE OF SYNCHRONIZING BUS).

FIG. 80 - FOUR MAIN BUS ARRANGEMENT WITH TWO LOCATIONS.
protection for feeder circuits will be considered.

a. Single Channel Feeder

A single channel radial feeder may use overcurrent protection as shown in Fig. 81a. Co-ordination between overcurrent protective devices in the load circuits and feeder circuit must be closely checked to insure proper operation under all types of faults. In some cases overcurrent protection of a single channel radial feeder will not be adequate as the available normal and short circuit currents vary between wide limits when both the load and sources are switched on and off. This is particularly true on long lines where there may be instances when the maximum short-circuit current under some conditions may approach the normal load current under other conditions. For this case other forms of protection such as the three-phase two-wire differential current scheme previously described may be desirable (Fig. 81b). This differential scheme also has the advantages of being faster and more responsive to low currents than is overcurrent protection.

Another type of protection for single channel feeders using balanced current protection is shown in Fig. 81c. This method should also be fast and sensitive. Detection of short circuits near the load bus with this scheme is the most difficult problem and it would probably be necessary to install reactors at the load bus. Balanced current protection may have the advantage of a high order of protection with out the disadvantage of pilot wires as required for differential current.

b. Double Channel Feeders

Double channel feeder service requires switching equipment at both the load and source ends of each feeder. The feeders may be supplied
FIG. 81 - SINGLE CHANNEL FEEDERS

A - OVERCURRENT PROTECTION  B - DIFFERENTIAL PROTECTION  C - BALANCED CURRENT PROTECTION
from the same main bus although it is preferable that each channel be supplied from a separate main bus. This latter scheme requires that the main tie system be reliable and have low impedance, since it is undesirable that transfer currents between main buses pass through the load buses. Several arrangements of multiple channel service are covered in the following paragraphs.

1) **Overcurrent Protection - Fig. 82**

   Multiple channel service with overcurrent protection supplied from two main buses must have the feeder overcurrent devices carefully coordinated with both the load and tie circuit protection. Note in the figure that in the event of a main bus fault it is likely that the tie protective devices will be tripped first. Then the fault is fed through the feeder circuits and it is likely that the channel connected to the good bus will have its protective device tripped first leaving the load buses connected to the faulted main bus without useful power. This difficulty is of course avoided where the main bus is provided with fast protection such as differential current which will trip the feeder as well as the source and tie circuits. Hence, Fig. 82a is applicable only where circuit breakers are used at the source end and should not be used with fuse protection. Fig. 82b can be used regardless of the form of main bus protection.

2) **Overcurrent and Directional Protection - Fig. 83a**

   In this arrangement the directional devices at the load bus are made fast compared to the overcurrent devices at the sending end of the line. Present evidence from the study of directional protection indicates that adequate directional protection would be obtained for all faults excepting low impedance three phase faults. It now appears probable that the
TIE SYSTEM

MAIN BUS

MAIN BUS

4 OR MORE
CHANNELS

LOADS

A - OVERCURRENT PROTECTION
SERVICE FROM TWO MAIN BUSES.

LOADS

B - OVERCURRENT PROTECTION
SERVICE FROM ONE MAIN BUS.

FIG. 82 - MULTIPLE CHANNEL FEEDER CIRCUITS.
directional device should sense reactive power to give the best sensitivity to short circuits. Application of this form of protection requires consideration of selectivity between the overcurrent devices in the feeder and load circuits. The effects of directional flow from motor loads must also be considered.

3) **Differential Current Protection - Fig. 83b**

Since differential current protection senses faults only in its own protected zone it can be used in any combination of feeder circuits. Fig. 83b shows its use in dual channel feeder service. This scheme has the advantage of being the easiest system to apply. Its disadvantage is the necessity of using pilot wires. In spite of this disadvantage it is believed to have considerable merit for aircraft application. In the arrangement shown a direct acting trip on each circuit breaker is contemplated.

4) **Balanced Current Protection - Fig. 83c**

Balance current protection as previously explained for single channel service is also applicable to dual channel service as shown in Fig. 83c.

5) **Emergency Throwover Arrangements - Fig. 84**

Various emergency throwover schemes are possible. The arrangement shown in Fig. 84 has previously been suggested. Arrangements of this type should be satisfactory. It is considered desirable that the throwover be automatic and not require operator attention. The chief disadvantages of such a system are:

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1 Fig. 1, Page 12, Exhibit MOREXE 21-72
FIG. 84 DUAL CHANNEL FEEDER CIRCUIT
EMERGENCY THROWOVER METHOD.
(a) No advantage is taken of the alternate channel to obtain better voltage regulation under normal conditions.

(b) Failure of the contactor on the load end is not self indicating and may remain in an unusable condition between overhaul checks.

(c) Contactor operates each time system is energized. Thus there is some danger of the contacts becoming welded in a closed position during this operation on fault conditions. Again such a failure would not be self indicating.

c. **Summary**

It is believed that the two arrangements which show the most promise for early development are:

1. Differential current
2. Combination overcurrent and directional current.

The former has the outstanding advantage of freedom from co-ordination problems with the rest of the system. Differential protection can also be used with several buses in sequence and permit more economical distribution system arrangements. The disadvantages of differential protection are:

1. Pilot wires
2. Other forms of load bus protection must be provided.

Combination overcurrent and directional protection avoid these two difficulties but do introduce co-ordination and time delay problems.

It is believed that development of suitable protective devices using the above methods should be initiated and vigorously pursued. These devices are essential to obtaining a reliable trouble free a-c electric power system.

4. **Comparison Between Fuse Mesh Network and Differential Current Protected Bus and Tie Line**

Fuse mesh networks have been used in aircraft a-c electrical sys-
tems. The major advantage of such systems is that they provide several possible channels for current flow. Their chief disadvantage is their relative complexity particularly where they are associated with remote electrically operated switching circuits. One ring bus arrangement which has been proposed is shown in Fig. 85 in a three line diagram.¹

The previous discussion in this section emphasized the simplicity which other methods of protection would lend to the bus and tie line system. To illustrate this fact a three line diagram of the power circuits of a differential current protected system is shown in Fig. 86. It is believed that the differential current protected system will offer the following advantages over the fuse mesh system:

1. Ease of operation
2. Ease of maintenance
3. Faster protection
4. Freedom from co-ordination problems
5. Lower weight
6. Less exposure of power system.

¹ Fig. 1, Page 12, Exhibit MOREXE 21-72
FIG. 85 - 3 LINE DIAGRAM OF RING BUS WITH FUSE MESH PROTECTION

FORWARD LOADS

AFT LOADS

ALT 4

ALT 3

RIGHT WING LOADS

BT IE

BT E3
FIG. 66—THREE LINE DIAGRAM OF RING DIFFERENTIAL PROTECTION.
DIAGRAM OF RING BUS WITH OVERCURRENT PROTECTION.
VI. GENERATOR CONTROL SYSTEM

The controls for the a-c generators are shown in Fig. 1 and Fig. 2. The exciter is switched separately from the main circuit breaker which connects the generator to the system. Manual synchronizing is used.

A serious disadvantage of the 400-cycle a-c system is the relative complexity to the operator. In the d-c aircraft systems the operator has to operate a single on-off switch for each generator. From there on operation is automatic. (Some systems incorporate a manual voltage control although this is not considered essential and, in fact, may be undesirable).

There appears to be advantage in automatic operation of an a-c generator control system following closing of the operator's control switch. Some consideration has been given to such a system and one approach to the problem will be discussed. A schematic diagram of the arrangement is shown in Fig. 87.

In this arrangement the operator's control and protective switching functions are carried out by separate devices. With this separation of functions, several advantages worthy of consideration appear.

The circuit breaker need not be designed for repetitive operation and since it will be operated only in the event of a short circuit, it should be capable of interrupting the short-circuit current. In many cases the circuit breaker could be a manually closed device, in fact there may be safety advantages in using a circuit breaker that can only be closed manually.

The contactor need not be made suitable for interrupting full available fault current, although there is some advantage in having the higher interrupting ability. Then the protective relays may be made to operate the
B - PROTECTIVE CIRCUIT BREAKER
C - CONTACTOR, 3 POLE AUXILIARY CONTACTS 3NO, 3NC
V - VOLTAGE RELAY
$V_B$ - VOLTAGE RELAY
SC - SYNCHRONISM CHECK RELAY
M - SHADED POLE MOTOR WITH GEAR DRIVE
   FOR VERNIER SPEED CONTROL
R - VERNIER SPEED CONTROL RHEOSTAT
CS - CONTROL SWITCH SPST (ON-OFF)

FIG. 87 - AUTOMATIC A-C GENERATOR CONTROL SYSTEM
contactor as well as the circuit breaker. In this way, even though tripping power were not available for the main circuit breaker during a fault, the circuit would be cleared.

Operation of the control circuit is described as follows:
At all times, except when the protective devices indicate "trip", the exciter field circuit and the main circuit breaker are closed. After starting an engine, the generator voltage builds up as engine speed increases until the generated voltage reaches the normal level as set by the voltage regulator. As the generated voltage approaches normal level relay V picks up. This places switching of the contactor C under control of the operator control switch CS and the synchronism check relay SC.

During this period the frequency of the incoming generator is determined by the setting of a vernier rheostat, R, in the governor circuit. The speed will increase to a value determined by the then existing setting of the rheostat. When V and CS are both closed, regardless of which occurred first, (the operator may close the control switch at any time before or after the engine is started) the small shaded-pole motor M is energized. This motor drives the vernier speed control rheostat so as to alternately increase and decrease the speed of the incoming machine from normal as the rheostat is continuously rotated through 360 degrees. The vernier rheostat may be a variable ratio transformer if this is advantageous in obtaining the plus-minus speed control or in securing long life. The motor M may operate for some time if the engines are run at subnormal speed during ground operation.

When the proper frequency and phase relationships are reached the synchronizing check relay SC will close to energize the contactor C which
seals itself in. When contactor C operates, the alternator is connected to the bus, the speed vernier drive-motor M is de-energized and the speed vernier rheostat shorted out leaving the load setting as determined by the normal speed adjusting rheostat.

In order to connect the first machine to a dead bus, another voltage relay, $V_B$, is used. When $V_B$ is energized, the $V_B$ contacts in series with contacts of $V$ are closed, permitting the sequence of operations previously described. If $V_B$ is not energized (as when the bus is dead) contacts $V$ and $SC$ are shorted out by the normally closed contacts of $V_B$ permitting contactor C to be energized directly by operating switch CS.

Obviously, considerable study needs to be given to the characteristics of the synchronism check relay, particularly with regard to pickup values and speed of operation. These in turn will be related to the speed and range of control of the vernier speed control driven by motor M.

The above control system has been sketched only in brief and several variations are possible.

Further study of the objectives of generator control systems for simplified operation should be carried out and techniques investigated for accomplishing them.
VII. CONCLUSIONS

1. Alternators having a lower short-circuit ratio than the present 40 KVA machines can probably be tolerated without endangering system stability.

2. Test methods of determining alternator characteristics should be studied and standardized.

3. The present reactive current amplification factor gives a loss of 11% in capacity on a four-alternator system due to an error voltage of +2.5%.

4. With the present frequency governing system using the equalizer circuit and the maximum droop setting, the loss in system capacity in a four-alternator system is 16% due to an error of +5% in the reference frequency.

5. In case of loss of regulator sense on one alternator of a 2-alternator system, voltage may increase to 13% above normal and may persist until cleared by the exciter protection relay in about 2 seconds.

6. Overvoltage protection (i.e., protection against sustained overvoltage resulting from loss of regulator control) is presently accomplished only by the exciter protection relay. The exciter protection relay must be inherently slow to avoid nuisance operation on heavy load conditions.

7. Loss of field excitation results in large reactive power flow from remaining good machines and produces subnormal voltage. For continued successful operation of the system, it is essential to remove the underexcited alternator from the system.
8. Failure of speed control would result in overspeeds of all alternators connected to the system.

9. The available symmetrical rms fault currents from the 40 KVA alternators studied are as follows:
   a. Single-line-to-ground is 6 per unit.
   b. Line-to-line is 4 to 5 per unit.
   c. Three-phase is 6 1/2 per unit.

10. If the rotor of an alternator becomes blocked, it imposes a severe drain of real and reactive power on the system and results in a reduced system voltage. Automatic means of disconnecting the faulty machine are essential.

11. Synchronizing disturbances may give rise to reduced system voltage and to excessively high currents.

12. On a single alternator system, with overvoltage protection provided only by the exciter protection relay, load equipment would need to be designed to withstand approximately 160 volts (133 percent) line-to-neutral for a duration determined by the speed with which the exciter protection relay operates — 2 seconds or more. All other overvoltage conditions found were much less severe for overheating effects.

13. Both differential and directional protection are desirable for alternator protection.

14. Protection is needed to remove an alternator which has lost its field excitation. Directional reactive protection would be desirable providing methods of discriminating between loss field excitation and voltage sense can be determined.
15. Differential protection using the double current transformers per phase with two pilot wires for all three phases appears to offer possibilities for protection of aircraft alternators and leads to the bus.

16. Differential current protection for main buses is desirable.

17. Differential current or combinations of directional and overcurrent devices appear to offer possibilities for distribution feeder protection.
VIII. RECOMMENDATIONS

1. Studies should be carried out to determine acceptable short-circuit ratios for aircraft alternators.
   b. Minimum acceptable values for system stability.

2. Studies should be initiated to establish standardized test procedures for determining alternator characteristics.

3. The values of pertinent alternator time constants should be determined for future analytical studies.

4. Economic studies should be made to determine the optimum value for both the reactive power equalizer and the real power equalizer amplification factors.

5. Methods of reducing the temperature effects on regulated frequency should be investigated.

6. Investigations should be carried out to determine better and faster methods of protection against overvoltages caused by loss of regulator control.

7. There is need for additional study of possible methods to remove the underexcited alternator from the system on loss of field excitation.

8. Additional investigations should be carried out to determine the effects of loss of frequency control.

9. Additional investigations should be carried out to determine the effects of failures in the reactive load and real load equalizer circuits.
10. Automatic means of detecting a blocked rotor should be developed.
11. Methods of automatic synchronizing should be developed.
12. Studies of transient overvoltages should be made.
13. Aircraft electrical systems should be designed with the following statements as guides
   (a) Failure of any device or circuit may occur.
   (b) The system should be so arranged that it will continue to operate successfully in spite of such a failure.
14. Arcing, bouncing, solid, pinched and any other faults should be further investigated to determine characteristics which can be utilized for system protection.
15. Specification MIL-F-5372 should be revised to specifically define the interrupting rating in terms of available fault current.
16. Fuse application should be based on using a multiplying factor of 1.4 to account for the d-c offset.
17. MIL-F-5372 should be revised to specify accuracy of melting characteristics down to melting times representative of the maximum short-circuit currents but not less than 0.0006 seconds.
18. Additional studies should be given to directional reactive protection for alternators. (This study is closely related to that suggested in Recommendation 6).
19. All elements of a unit system, a failure in any one of which would cause the loss of the same system component, should be arranged physically so that the exposure area is a minimum. For example, if the control leads of an alternator are run widely separated from
the main power leads, there is an increase in the vulnerable area.

20. Improved differential protection devices should be developed for application to alternators, buses, tie circuits, and feeders.

21. Overcurrent and directional protective devices should be developed for feeder service.

22. Studies should be directed toward improved control systems which are more nearly automatic.

23. The speed-sensing tachometer and overspeed limit switch should not be operated from the same shaft and gears, since a failure of either the shaft or gears would cause the simultaneous loss of both the speed controlling and protective devices.

24. A study should be made to determine whether the rotating exciter can be eliminated in favor of a magnetic amplifier-rectifier system.
IX. LOGBOOKS

Detailed data are recorded in Armour Research Foundation Logbooks C-1718, C-1719, C-1975, D-1124, and C-1986.